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Developed Macro-Scale Flow and Heat Transfer in Micro-Channels with Large Arrays of Offset Strip Fins for a Uniform Heat Flux

Arthur Vangeffelen[*1,2,3], Geert Buckinx[1,2,3], Maria Rosaria Vetrano[1,3], Martine Baelmans[1,3]
[1]Department of Mechanical Engineering, KU Leuven, Celestijnenlaan 300A, 3001 Leuven, Belgium
[2]VITO, Boeretang 200, 2400 Mol, Belgium
[3]EnergyVille, Thor Park, 3600 Genk, Belgium
*Corresponding author: arthur.vangeffelen@kuleuven.be (A. Vangeffelen)

Abstract **-** In this paper, we assess the macro-scale modelling of the flow and heat transfer in micro-channels with large arrays of periodic offset strip fins. This modelling approach is based on a double volume-averaging operation, allowing for an exact reconstruction of the macro-scale variables in the periodically developed flow and heat transfer regime. In particular, the approach is applied to micro-channels with an array of offset strip fins subject to an imposed uniform heat flux, for which the developed closure models are illustrated through friction factor and Nusselt number correlations. The validity of the developed closure models is analyzed by using direct numerical simulation to evaluate the flow and temperature in various micro-channels containing an array of offset strip fins.

Keywords**:** Micro-Channels, Macro-Scale Modelling, Closure, Periodic Flow and Heat Transfer, Offset Strip Fin Array

1. Introduction

Micro-channels with large arrays of periodic solid structures exist in many applications [1, 2]. A growing interest in these channels exists because of the continuous evolution of compact heat transfer devices towards smaller scales [3]. A common fin geometry is the offset strip fin, employed in applications such as micro-HVAC systems, compact gas turbine recuperators, coolers of microelectronic devices and solar air heaters [4, 5, 6]. For these applications, the flow typically remains in the laminar regime, and the thermal boundary condition of the heat transfer regime can be approximated by a uniform heat flux supplied to the channel wall [7]. To characterize the thermo-hydraulic performance as a function of the mass flow rate for offset strip fin micro-channels with an imposed uniform heat flux, only a unit cell of the array is commonly considered under the assumption of periodically developed flow and heat transfer. In contrast to direct numerical simulations, this significantly reduces the required computational resources.

A first unit cell approach is the volume-averaging technique for porous media reported by Whitaker [8, 9]. This approach allows predicting the pressure gradient and heat transfer coefficient for a given mass flow rate by solving the closure equations for the deviation component of the velocity, pressure and temperature fields. A second approach is to compute the pressure gradient and heat transfer coefficient through the periodically developed flow and heat transfer equations from Patankar [10], which govern the periodic component of the velocity, pressure and temperature fields.

If a double volume-averaging operation is selected over the unit cell to define the macro-scale variables, the closure models of Whitaker and the periodically developed equations of Patankar become equivalent and exact. This macro-scale description was introduced by Buckinx and Baelmans [12, 13] by building on the work of Quintard and Whitaker [11]. In this framework, the macro-scale variables become uniform and physically meaningful. However, it is still theoretically unknown when this macro-scale description for the periodically developed flow and heat transfer regime is valid for common applications of micro-channels with arrays of offset strip fins.

In this work, we analyze the developed friction factor and Nusselt number for offset strip fin micro-channels subject to an imposed uniform heat flux. This analysis is done in a macro-scale framework based on the double volume-averaging operation by solving the periodically developed closure equations on a unit cell. Moreover, we assess to which extent these closure models for developed flow and heat transfer are valid through direct numerical simulation of the flow and heat transfer in an entire offset strip fin array.

2. Developed Macro-Scale Flow and Heat Transfer

When the macro-scale variables are defined through a double volume-averaging operation $\langle\ \rangle_m$, these variables become spatially constant and physically meaningful in the periodically developed flow and heat transfer regime. In this regime, the macro-scale velocity $\langle \boldsymbol{u} \rangle_m$ becomes uniform and equal to the volume-averaged velocity $\langle \boldsymbol{u} \rangle$, which is related to the volumetric flow rate, and the macro-scale pressure gradient $\nabla \langle p \rangle_m^f$, corresponding to the macro-scale no-slip force $\boldsymbol{b}_{fs}$, becomes uniform and equal to the constant overall pressure gradient ∇P over the unit cell:

$$\langle \boldsymbol{u} \rangle_m = \langle \boldsymbol{u} \rangle, \text{ and } \nabla \langle p \rangle_m^f = \varepsilon_{fm}^{-1} \boldsymbol{b}_{fs} = \nabla P, \tag{1}$$

with $\varepsilon_{fm} \triangleq \langle \gamma_f \rangle_m$ the weighted porosity of the unit cell and γ_f the fluid indicator function: $\gamma_f = 1$ in the fluid domain and $\gamma_f = 0$ in the solid domain. Moreover, for periodically developed heat transfer driven by an imposed uniform heat flux q_b, the temperature field T can be decomposed into a component that varies linearly with a spatially constant gradient ∇T and a spatially periodic component T^* [10]. Consequently, also the macro-scale interfacial heat transfer coefficient h_{fs} becomes spatially constant and corresponds to the volume-averaged imposed heat flux $\langle q_b \delta_b \rangle$:

$$h_{fs} \triangleq \varepsilon_{fm}^{-1} \frac{\langle q_{fs} \delta_{fs} \rangle_m}{\langle T \rangle_m^f - \langle T \rangle_m^s} = \varepsilon_{fm}^{-1} \frac{\langle q_b \delta_b \rangle}{\langle T^* \rangle^f - \langle T^* \rangle^s}. \tag{2}$$

Therefore, a relationship between $\boldsymbol{b}_{fs}$, h_{fs} and $\langle \boldsymbol{u} \rangle_m$ can be determined for a given unit cell geometry. For example, for micro-channels with an array of offset strip fins, this can be represented in a non-dimensional form by performance correlations between the friction factor $f_{unit} \triangleq \|\nabla P\| l / \left(2\rho_f \|\langle \boldsymbol{u} \rangle\|^2\right)$, the Nusselt number $Nu_{unit} \triangleq h_{fs} l^2 / k_f$ and the Reynolds number $Re_l \triangleq \rho_f \|\langle \boldsymbol{u} \rangle\| l / \mu_f$. The friction factor correlation can be found in our previous work [14], whereas the following empirical correlation for the Nusselt number has been obtained through a least-squares fitting procedure:

$$Nu_{unit} = c_0 + c_1 Re_l, \text{ with} \qquad \begin{aligned} c_0 &= 21.2(h/l)^{-1.59} + 23.3(s/l)^{-1.85}, \\ c_1 &= 1.27(s/l - t/l)^{-1.02}(t/l)^{0.56}(h/l)^{-0.63}. \end{aligned} \tag{3}$$

Here, the geometrical parameters of the offset strip fin unit cell are the fin thickness t, the fin height h, the lateral fin pitch s, and the fin length l. These correlations have been constructed over the ranges $Re_l = 1 - 600$, $h/l = 0.12 - 1.00$, $s/l = 0.12 - 0.48$, $t/l = 0.01 - 0.06$, with $Pr_f \triangleq \mu_f c_f / k_f = 7$ and $k_s/k_f = 500$. In the following sections, the validity of the developed closure correlations will be assessed for entire arrays of offset strip fins in typical micro-channel applications.

3. Onset of Developed Macro-Scale Flow

To know where the closure models are valid, the extent of the periodically developed region in an entire micro-channel needs to be quantified for a given bulk average velocity u_b through the channel. The onset of the periodically developed region can be determined by analyzing the velocity profile as a function of the streamwise position in the channel x_1. In Figure 1, it is illustrated that after the development length x_{dev} from the inlet section x_{in}, the macro-scale velocity $\langle \boldsymbol{u} \rangle_m$, becomes spatially constant, corresponding to the periodically developed flow regime. A linear relationship has been observed between x_{dev} and the Reynolds number $Re_b \triangleq \rho_f u_b l / \mu_f$, as illustrated in Figure 2 for different geometries. It is clear that the development lengths remain relatively small for offset strip fin arrays in micro-channels. Due to the small values of x_{dev}, the discrepancies between the developed closure model in [14] and the local macro-scale no-slip force are found to remain below 10% in the inlet region of the channel. Therefore, the correlation in [14] can accurately predict the pressure drop through the offset strip fin array for the developed flow region as well as a large part of the inlet region. Furthermore, for all cases considered in this work, the influence of the channel's outlet was found to remain limited to the streamwise geometry period $l_1 = 2l$, whereas the channel's side-wall influence remains limited to the lateral geometry period $l_2 = 2(s+t)$.

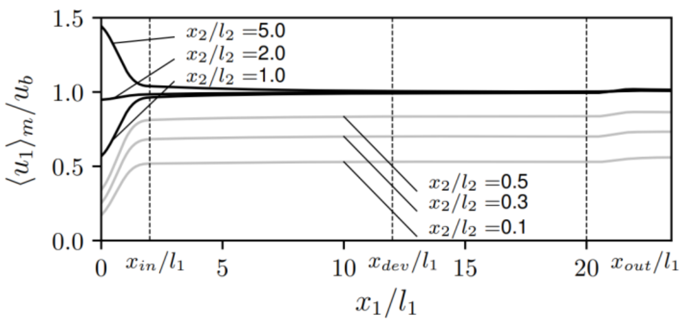

Figure 1: Macro-scale streamwise velocity component inside (black) and outside (grey) the channel's side-wall region

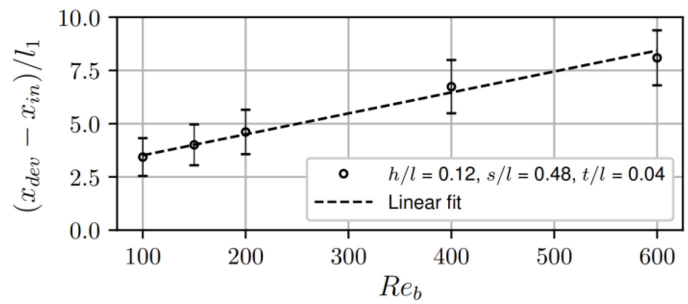

Figure 2: Influence of the Reynolds number on the flow development length

Near the side-wall of the channel, a porosity gradient exists and the macro-scale flow is decreased as the flow is no longer periodically developed. This results in a two-dimensional macro-scale velocity profile $\eta \triangleq \|\langle \boldsymbol{u}\rangle_m\|(x_2)/u_b$ as a function of the lateral position x_2, which can be recognized by the light grey lines in Figure 1. It has been observed that this profile is independent of the Reynolds number for a given array geometry. Furthermore, this profile can be fitted by a linear function determined by a single parameter: the so-called slip length, which corresponds to the fact that the macro-scale velocity has a slip boundary condition at the channel's side-wall. Through η, the effects on the mass flow rate can be determined and the macro-scale no-slip force can be exactly reconstructed in the side-wall region. Due to the limited extent of the side-wall influence, η can be obtained by solving the flow equations on an extended unit cell containing two unit cells. One is part of the periodically developed region, and the other is adjacent to the channel side-wall.

4. Onset of Developed Macro-Scale Heat Transfer

In analogy to the flow regime, the onset of the periodically developed heat transfer regime can be determined by analyzing a temperature profile along the streamwise direction. Figure 3 illustrates that after the development length $x_{dev,T}$ from the inlet section x_{in}, also the macro-scale periodic temperature $\langle T^*\rangle_m$ becomes spatially constant, corresponding to the periodically developed heat transfer regime. Similar to the flow development length, the relationship between $x_{dev,T}$ and Re_b can be captured by a linear fit, as illustrated in Figure 4. Again, since the development lengths remain limited for the considered cases, the discrepancies between the developed closure model (3) and the local macro-scale interfacial heat transfer coefficient remain below 15% in the inlet region of the channel. Also for the heat transfer regime, the outlet and the channel's side-wall influence are limited to $l_1 = 2l$ and $l_2 = 2(s+t)$, respectively.

5. Conclusion

In this work, the developed macro-scale flow and heat transfer in micro-channels with an array of offset strip fins has been analyzed. The macro-scale description is based on a double volume-averaging operation, resulting in spatially constant and physically meaningful macro-scale variables in the periodically developed regime. In this framework, closure models were presented for arrays of offset strip fins in typical micro-channel applications, corresponding to correlations for the friction factor and Nusselt number as a function of the Reynolds number and the offset strip fin geometry. Next, the onset of the developed flow and heat transfer regime was observed to move downstream in the channel as the Reynolds number increases, where both the flow and heat transfer development length depends linearly on the Reynolds number. Nevertheless, for both the flow and the heat transfer regime, these development lengths were proven to remain limited.

Consequently, for offset strip fin arrays in micro-channels, the developed closure models can still accurately predict the inlet region of the channel.

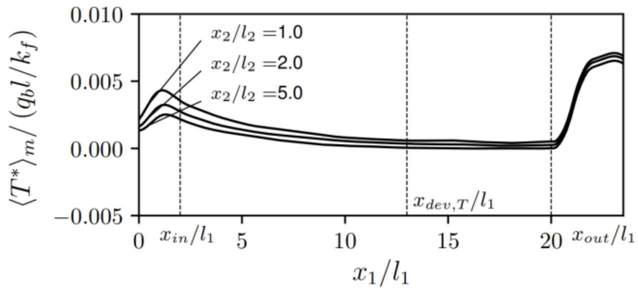

Figure 3: Macro-scale periodic temperature component

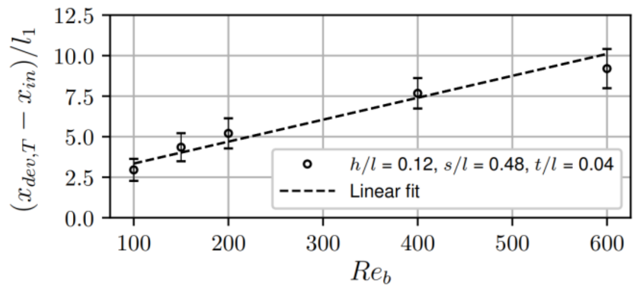

Figure 4: Influence of the Reynolds number on the heat transfer development length

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