Proceedings of the 7th World Congress on Momentum, Heat and Mass Transfer (MHMT'22)
~~Lisbon, Portugal~~ Virtual Conference – April 07 – 09, 2022
Paper No. ENFHT 240
DOI: 10.11159/enfht22.240

Numerical Analysis of Heat Transfer within Two Anisotropic Coaxial Mediums in Cylindrical Geometry

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Abstract - In this paper, the unsteady numerical study of two anisotropic cylindrical mediums in two-dimensional configuration space is investigated. The left and the right sections of the cylinders are presumed to have imposed temperatures, while a radial imposed heat flux has been exercised to the whole lateral surface of the outer medium. The analytical resolution of this genre of mediums in cylindrical configuration is tough as a consequence of the presence of the second-order derivation terms in the thermal heat equations. Thus, a linear coordinate transformation is utilized so as to transform the anisotropic equations to an equivalent isotropic equations form. The method of alternating directions implicit finite-difference (ADI) is adopted to integrate the governing equations of the two mediums numerically. The numerical results are obtained by a digital code which is developed and successfully confirmed. The numerical results presented in the form of a temperature profile showed that the analysis parameters, specifically thermal conductivities, anisotropy and the imposed radial flux have an effect on the shape and the thermal level inside the two anisotropic mediums.

Keywords**:** Two anisotropic mediums, cylindrical configuration, unsteady numerical study, linear coordinate transformation, main and cross thermal conductivities, ADI.

1. Introduction

In mechanical engineering and applied science, the study of anisotropic materials has been of great significance for the fields of aerospace, automobile engineering and heat exchangers. This kind of materials is designed to meet new needs whether in terms of mechanical, chemical or thermal resistance which is the case that interests us.

The study of the thermal behaviour of this kind of materials leads to partial differential equations which contain the second-order derivative terms, these terms make solving the anisotropic equations difficult. The orthotropic mediums characterized by the absence of this term in the governing equation, many studies have been investigated in this case [1-2], among others, Haji-Sheikh et al [1] who has used the separation of variables method as a means to find a solution of the transfer heat equation.

In the case of anisotropic mediums, the studies are rare and as far as we know, most of this works treat only the case of two-dimensional steady state heat conduction [3-7] and transient heat transfer regime [8-9]. For heat transfer in a three-dimensional anisotropic configuration, the works are referred to [10-11]. Based on the fact that there is no analytical solution to the equations of thermal heat conduction in anisotropic mediums where the configuration is finite, a numerical study of the thermal behavior of two anisotropic mediums is at the core of this research.

The work targets to examine the effect of the anisotropy on the heat transfer and the necessary time to obtain the steady state conditions.

2. Mathematical Model

The configuration studied is two anisotropic cylindrical mediums A and B of length L and two rays b_1 and b_2 respectively as demonstrated in Figure 1. The two ends of the two mediums are maintained at constant temperatures (T_L and T_R), whereas a radial flux is applied to the lateral surface of the outer medium B.

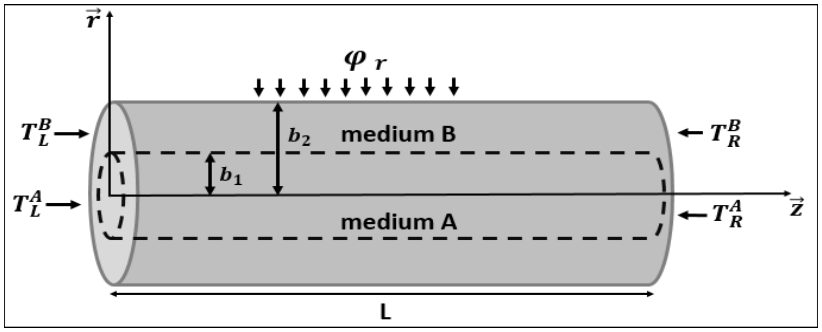

Figure 1: Geometry and boundary conditions of the solid cylinders.

By taking into account the symmetry of the two cylinders, the thermal heat conduction equations for the two anisotropic mediums in the plane (r,z) are expressed by [12]:

$$K_r^i \frac{\partial^2 T^i}{\partial r^2} + 2K_{rz}^i \frac{\partial^2 T^i}{\partial r \partial z} + K_z^i \frac{\partial^2 T^i}{\partial z^2} + K_r^i r \frac{\partial T^i}{\partial r} + \frac{K_{rz}^i}{r} \frac{\partial T^i}{\partial z} = \rho_i C_p^i \frac{\partial T^i}{\partial t} \quad (1)$$

$$\begin{cases} i = A \quad for \quad medium \quad A \\ i = B \quad for \quad medium \quad B \end{cases}$$

The radial and axial heat fluxes for the two mediums are:

$$\begin{cases} \varphi_r^i = K_r^i \dfrac{\partial T^i}{\partial r} + K_{rz}^i \dfrac{\partial T^i}{\partial z} \\ \varphi_z^i = -K_{rz}^i \dfrac{\partial T^i}{\partial r} - K_z^i \dfrac{\partial T^i}{\partial z} \end{cases} \quad (2)$$

With K_r^i, K_z^i are the main thermal conductivities and K_{rz}^i the cross thermal conductivities for the two mediums. ρ and c_P are respectively density and specific heat and t is the time.

The above equations are allied to the following boundary and interface conditions:

$$0 \le z \le L \quad r = 0 \qquad \frac{\partial T^A}{\partial r} = 0$$

(3)

$$0 \le z \le L \quad r = b_1 \qquad \begin{cases} T^A(r\ ,z\ ) = T^B(r\ ,z\ ) \\ \varphi_r^A(r\ ,z\ ) = \varphi_r^B(r\ ,z\ ) \end{cases} \quad (4)$$

$$0 \le z \le L \quad r = b_2 \qquad \varphi_r = K_r^B \frac{\partial T^B}{\partial r} + K_{rz}^B \frac{\partial T^B}{\partial z}$$

(5)

$z=0$ *and* $z=L$

$$\begin{cases} T^A(r,0)=T^B(r,0)=T_L \\ T^A(r,L)=T^B(r,L)=T_R \end{cases} \tag{6}$$

3. Results and Discussions

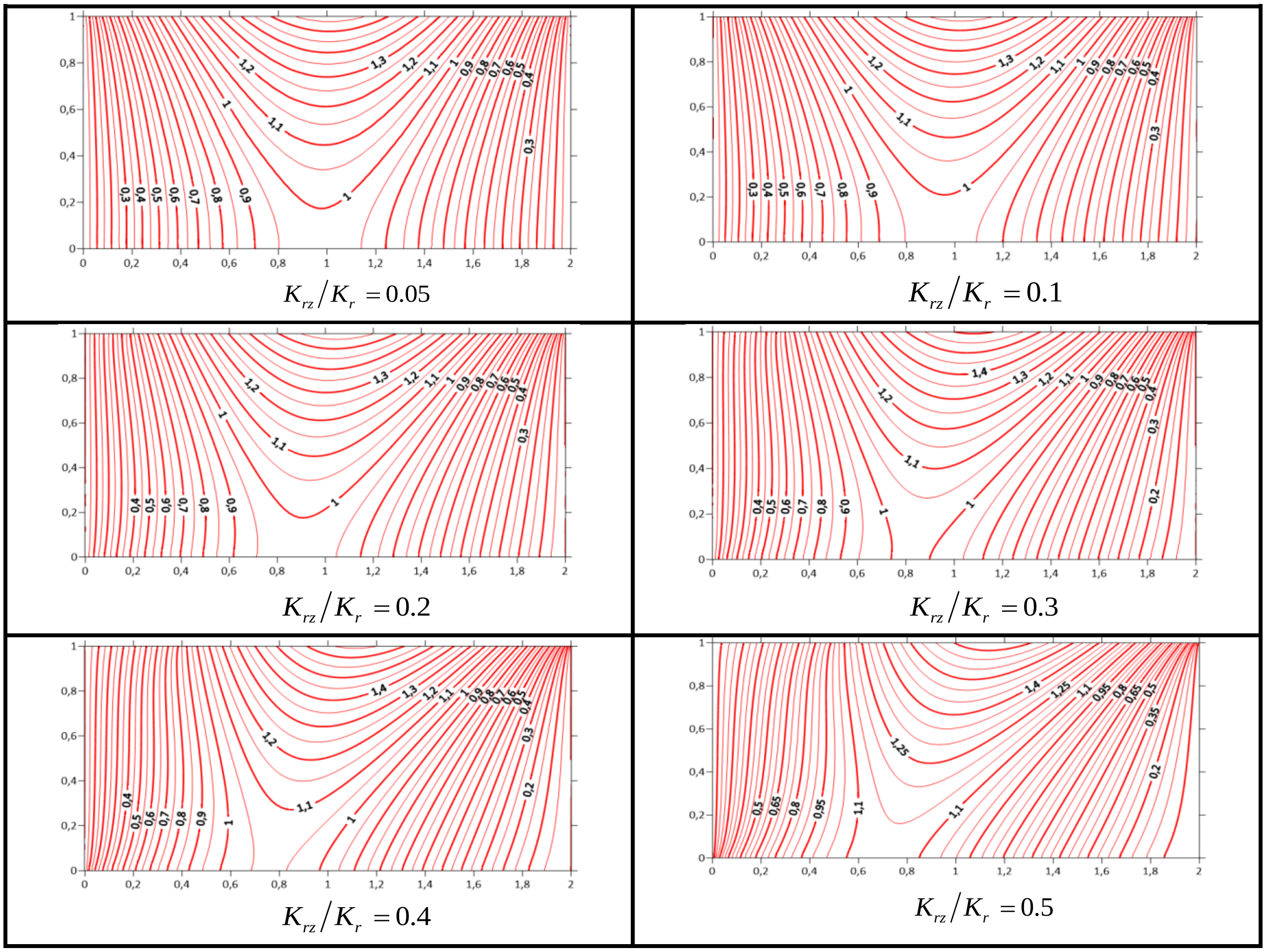

Figure 2: Steady state temperature profiles for $K_z/K_r = K_r^B/K_r^A = \varphi_r^* = 1$

The resolution of the thermal heat conduction equations within the two anisotropic mediums in cylindrical configuration is done by a numerical code that provides access to the mediums dimensionless temperature profiles $\theta^i(r,z)=(T^i(r,z)-T_R)/T_R$ and the necessary time $t^* = t.\alpha_r^B/b_2^2$ (α_r^B being medium B thermal diffusivity) to obtain the steady state conditions. For a better understanding of the impact of the anisotropy on the thermal behaviour inside the two mediums, the ratio K_{rz}/K_r which represents the anisotropy was chosen equal to 0.05, 0.1, 0.2, 0.3, 0.4, 0.5.

The numerical results presented in figure 2 are the dimensionless temperature profiles $\theta^i(r,z)=(T^i(r,z)-T_R)/T_R$ inside the two anisotropic mediums in the real space (r,z) for the above values of K_{rz}/K_r and for main thermal conductivities ratios $K_z / K_r = 1$ and $K_r^B / K_r^A = 1$ and for imposed radial flux equals 1 ($\varphi_r^* = L.\varphi_r / (K_z^B.T_R) = 1$). The dimensionless temperatures applied on the two sections of the two mediums are taken equal to 0 ($\theta_L^i = \theta_R^i = 0$).

Proceeding from the distribution of the temperatures illustrated in figure 2, it is obvious that the Isotherms are affected by the anisotropy. An increase of this parameter breaches the symmetry presented for low values of K_{rz}/K_r and increases the thermal level of the isotherms. In addition to the previous results, Table 1 summarizes the required time to obtain the steady state regime for the given values of K_{rz}/K_r. An increase of the anisotropy increases this time.

Table 1: necessary time to obtain the steady state conditions.

K_{rz}/K_r	**0.05**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**
t^*	4.175	4.195	4.2	4.278	4.446	4.709

4. Conclusion

The influence of the anisotropy characterized by the ratio K_{rz}/K_r on the thermal behaviour within two anisotropic mediums was studied. This study leads to a numerical code that provides access to the steady state temperature profiles and the necessary time to obtain the steady state conditions. A significant increasing of the thermal level of the isotherms has been observed with the increase of the anisotropy. This increase breaks also the symmetry of the isotherms which was present for the low values of K_{rz}/K_r. In addition to the previous conclusions, an augmentation of the anisotropy increases the required time to reach the steady state conditions.

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