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A New One-equation Turbulence Model based on the Combined Standard *k-ε* and *k-ω* Turbulence Models for Benchmark Flow Configurations

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Abstract - A new one-equation turbulence model based on the two-equation standard *k-ε* and Wilcox's *k-ω* turbulence models is proposed and developed for validating the benchmark flow configurations. The desirable features of both these two-equation turbulence models are combined into the new one-equation turbulence model in the complete form without neglecting the third-order velocity gradient term than simply assuming that equal coefficients in the diffusion terms. This new one-equation turbulence model is used to simulate for different benchmark flow configurations including the flow over a flat plate at zero pressure gradient, the bump-in-channel flow, the backward facing step flow and the NASA wall-mounted hump separated flow. The numerical results are fully validated and compared with the results of the experimental dataset, the one- and two-equation turbulence models, and the high-accuracy NASA codes (i.e., CFL3D and FUN3D). The new one-equation turbulence model is proved to be more accurate when compared with the one-equation Wray-Agarwal and two-equation shear stress transport (SST) *k-ω* turbulence models for the benchmark flow configurations.

***Keywords*:** CFD, turbulence modelling, one- and two-equation turbulence models, third-order velocity gradients, benchmark flows

1. Introduction

Two-equation turbulence models are normally used two transport equations for modelling two variables (i.e., length and time scales) in a turbulent flow region. A transport equation for the computation of turbulent kinetic energy, k and another transport equation for the turbulent dissipation, ε, the turbulent specific dissipation rate, ω or the turbulent length scale, L are most commonly used for two-equation turbulence models, which are known as the typical *k-ε*, *k-ω* or *k-kL* turbulence model, respectively. The eddy viscosity is then calculated through the turbulent length scale obtained by the two transport equations.

By comparison, one-equation turbulence models directly solve the transport equation of the eddy viscosity, rather than the algebraic length scales, which have the advantage of high computational efficiency [1-3]. Due to the obvious advantage in simplicity and accuracy, one-equation turbulence models have attracted increasingly attention [3-6]. Based on the wide scientific and engineering applications, many research studies have focused on the transformation of two-equation turbulence models to one-equation turbulence models [2, 7]. Menter [2] proposed the transformation methodology of two-equation *k-ε* turbulence model to a one-equation turbulence model based on the assumption that there is a linear relationship between the turbulent shear stress and turbulent kinetic energy. It is worth noting that the coefficients in the diffusion terms are assumed to be equal in the process of the transformation, while they are not the same in the original formulations, therefore neglecting some specific terms in the derivational process. Following the assumption, Han et al. [7] proposed and improved the one-equation turbulence model derived from the two-equation *k-ω* turbulence model by including a cross diffusion term which made it possible to change the behaviour of the one-equation turbulence model between the two-equation *k-ε* and *k-ω* turbulence models. The results showed that the one-equation turbulence model has a good agreement with the experimental data. Note that all these one-equation turbulence models based on the two-equation *k-ω* turbulence models do not include the third-order velocity derivative-based length scale.

It is common practice to assume equal coefficients of the diffusion terms in the parent equations in many research studies due to simplicity [2, 7]. However, a number of simplifying assumptions lead to neglecting several diffusion terms in the process of transformation of the parent two-equation turbulence model. The effect of these terms has not been fully examined and even for a very simple case, the performance of the resulting one-equation turbulence model may differ from the underlying parent two-equation turbulence model [2]. Based on the two-equation k-ε turbulence model, the one-equation turbulence model proposed by Elkhoury [1] does not assume equal coefficients of the diffusion terms leading to the emergence of the third-order velocity gradient term. The one-equation turbulence model is proved to be more accurate than other turbulence models due to the presence of the third-order velocity derivatives. Although the one-equation turbulence model retained the third-order velocity gradient term, a specific term related to the third-order velocity gradients is removed in the derivational process.

The present study is to derive the transport equations in the complete form and retain the third-order velocity gradient term without simply assuming equal coefficients of the diffusion terms in the process of transformation of the parent two-equation turbulence model to one-equation turbulence model. The newly proposed and developed one-equation turbulence model is combined the best characteristics of the two-equation standard k-ε (SKE) [8] and Wilcox's k-ω (WKO) [9] turbulence models. The accuracy of the new one-equation turbulence model is compared with the results of the experimental dataset, the commonly used one- and two-equation turbulence models and the high-accuracy NASA codes (i.e., CFL3D and FUN3D) for benchmark flow configurations.

2. Derivation of a New One-Equation Turbulence Model

The development of a new one-equation turbulence model based on two-equation SKE and WKO turbulence models is presented. The derived equations are written in boundary-layer coordinates for simplicity. x and y are represented the streamwise coordinate and normal to the boundary layer, respectively, and t is time.

2.1. Development of the One-Equation *k-ε* Turbulence Model

The two-equation SKE turbulence model in the boundary layer can be written as

$$\frac{Dk}{Dt}=\tilde{\nu}_t\left(\frac{\partial u}{\partial y}\right)^2-\varepsilon+\frac{\partial}{\partial y}\left[\left(\nu+\frac{\tilde{\nu}_t}{\sigma_k}\right)\frac{\partial k}{\partial y}\right] \tag{1}$$

$$\frac{D\varepsilon}{Dt}=C_{\varepsilon 1}\tilde{\nu}_t\left(\frac{\partial u}{\partial y}\right)^2\frac{\varepsilon}{k}-C_{\varepsilon 2}\frac{\varepsilon^2}{k}+\frac{\partial}{\partial y}\left[\left(\nu+\frac{\tilde{\nu}_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial y}\right] \tag{2}$$

where $\tilde{\nu}_t=C_\mu k^2/\varepsilon$ is the eddy viscosity, and ν is the kinematic viscosity. The time derivatives of k and ε are used to express the time derivative of $\tilde{\nu}_t$, a new transport equation for the turbulent viscosity is expressed as

$$\frac{D\tilde{\nu}_t}{Dt}=C_\mu\left(2\frac{k}{\varepsilon}\frac{Dk}{Dt}-\frac{k^2}{\varepsilon^2}\frac{D\varepsilon}{Dt}\right) \tag{3}$$

where σ_k, σ_ε, $C_{\varepsilon 1}$, and $C_{\varepsilon 2}$ are constants, and C_μ = 0.09. The relationship between k and $\tilde{\nu}_t$ is confirmed by many experimental boundary layer data [2, 7, 10], and the invariant value, S, is widely used to replace the absolute value of streamwise velocity gradient along the normal direction, which is expressed by

$$\left|-\overline{u'v'}\right|=\sqrt{C_\mu}k=\tilde{\nu}_t\left|\frac{\partial u}{\partial y}\right|=\tilde{\nu}_t S \tag{4}$$

where $\left|-\overline{u'v'}\right|$ is the turbulent shear stress and u is the streamwise velocity. Thus, a one-equation turbulence model is derived by straightforward substitution for preserving the completeness of SKE turbulence model, and the transport equation for $\tilde{\nu}_t$ in the complete form is solved as

$$\frac{D\tilde{\nu}_t}{Dt}=\beta_\varepsilon\tilde{\nu}_t S+\gamma_\varepsilon\frac{\tilde{\nu}_t}{S}\frac{\partial S}{\partial x_j}\frac{\partial \tilde{\nu}_t}{\partial x_j}+C_1\frac{\tilde{\nu}_t^2}{S}\frac{\partial}{\partial x_j}\left(\frac{\partial S}{\partial x_j}\right)-C_2\frac{\tilde{\nu}_t^2}{S^2}\frac{\partial S}{\partial x_j}\frac{\partial S}{\partial x_j}+\frac{\partial}{\partial x_j}\left[(\nu+\alpha_\varepsilon\tilde{\nu}_t)\frac{\partial \tilde{\nu}_t}{\partial x_j}\right] \tag{5}$$

It is worth noting that the third term on the right-hand side in Equation (5) involves the third-order velocity gradients, which is always neglected due to simplicity in many studies by assuming that the coefficients of the diffusion terms in the parent two-equation turbulence model are equal [2]. It is reported that the third-order velocity gradients are widely used to the combustion modelling due to their appearance in the thickened flame model [1, 11]. The values of the coefficients are recalibrated and given by $\alpha_\varepsilon = 1.0$, $\beta_\varepsilon = 0.144$, $\gamma_\varepsilon = 0.2386$, $C_1 = 0.0795$ and $C_2 = 1.5385$.

2.2. Development of the One-Equation *k-ω* Turbulence Model

The two-equation WKO turbulence model in the boundary layer can be written as

$$\frac{Dk}{Dt} = \widetilde{\nu}_t \left(\frac{\partial u}{\partial y}\right)^2 - \beta^* k\omega + \frac{\partial}{\partial y}\left[(\nu + \alpha_k \widetilde{\nu}_t)\frac{\partial k}{\partial y}\right] \tag{6}$$

$$\frac{D\omega}{Dt} = \gamma \frac{\omega}{k} \widetilde{\nu}_t \left(\frac{\partial u}{\partial y}\right)^2 - \beta\omega^2 + \frac{\partial}{\partial y}\left[(\nu + \alpha_\omega \widetilde{\nu}_t)\frac{\partial \omega}{\partial y}\right] \tag{7}$$

where $\widetilde{\nu}_t = k/\omega$ is the eddy viscosity. The substantial derivative of the eddy viscosity is expressed as

$$\frac{D\widetilde{\nu}_t}{Dt} = \frac{1}{\omega}\frac{Dk}{Dt} - \frac{k}{\omega^2}\frac{D\omega}{Dt} \tag{8}$$

where α_k, α_ω, β and γ are constants, and $\beta^* = 0.09$. By using a similar procedure with the derivation in Equation (4), the resulting one-equation based on WKO turbulence model can be written as

$$\frac{D\widetilde{\nu}_t}{Dt} = \beta_\omega \widetilde{\nu}_t S + \gamma_\omega \frac{\widetilde{\nu}_t}{S}\frac{\partial S}{\partial x_j}\frac{\partial \widetilde{\nu}_t}{\partial x_j} + \frac{\partial}{\partial x_j}\left[(\nu + \alpha_\omega \widetilde{\nu}_t)\frac{\partial \widetilde{\nu}_t}{\partial x_j}\right] \tag{9}$$

It is reported that the coefficient of the diffusion term is small, which leads to a large velocity gradient near the boundary layer edge [2] and a higher value is suggested [12]. The recalibrated coefficients are $\alpha_\omega = 1.2$, $\beta_\omega = 0.084$ and $\gamma_\omega = 1.7$.

2.3. Development of a New One-Equation Turbulence Model

Based on the two transport equations derived in Equations (5) and (9), Equation (5) is multiplied by $(1-F_1)$ and Equation (9) is multiplied by F_1, a completely new equation is then obtained as

$$\frac{D\widetilde{\nu}_t}{Dt} = \beta_\nu \widetilde{\nu}_t S + \gamma_\nu \frac{\widetilde{\nu}_t}{S}\frac{\partial S}{\partial x_j}\frac{\partial \widetilde{\nu}_t}{\partial x_j} + C_1(1-F_1)\frac{\widetilde{\nu}_t^2}{S}\frac{\partial}{\partial x_j}\left(\frac{\partial S}{\partial x_j}\right) - C_2(1-F_1)\frac{\widetilde{\nu}_t^2}{S^2}\frac{\partial S}{\partial x_j}\frac{\partial S}{\partial x_j} + \frac{\partial}{\partial x_j}\left[(\nu + \alpha_\nu \widetilde{\nu}_t)\frac{\partial \widetilde{\nu}_t}{\partial x_j}\right] \tag{10}$$

The coefficients in the Equation (10) are defined as

$$\alpha_\nu = \alpha_\omega F_1 + \alpha_\varepsilon(1-F_1), \beta_\nu = \beta_\omega F_1 + \beta_\varepsilon(1-F_1), \gamma_\nu = \gamma_\omega F_1 + \gamma_\varepsilon(1-F_1) \tag{11}$$

where F_1 is the blending function given by

$$F_1 = \tanh\left(\text{arg1}^4\right), \text{arg1} = \max\left(\frac{C_\mu^{1/4}\widetilde{\nu}_t^{1/2}}{\beta^* S^{1/2} d}, \frac{500 C_\mu^{1/2}\nu}{d^2 S}\right) \tag{12}$$

where d is the distance to the nearest wall. The newly developed one-equation turbulence model behaves like the two-equation SKE turbulence model when $F_1 \rightarrow 0$, while it functions as the two-equation WKO turbulence model when $F_1 \rightarrow 1$. The newly developed one-equation turbulence model combines the best features of these two-equation turbulence models. More importantly, the blending function has the similar form with that of the two-equation SST *k-ω* turbulence model [13], so the new one-equation turbulence model can behave like the two-equation SST *k-ω* turbulence model. To prevent the singularity when S goes to zero, it is necessary to bound S in the third term with a very small value while the fourth term of the right-hand side of Equation (10) is expressed as

$$\frac{\widetilde{\nu}_t^2}{S^2}\frac{\partial S}{\partial x_j}\frac{\partial S}{\partial x_j} = C_3 E_{BB} \tanh\left(\frac{E_{\widetilde{\nu}_t}}{C_3 E_{BB}}\right), E_{\widetilde{\nu}_t} = \frac{\widetilde{\nu}_t^2}{S^2}\frac{\partial S}{\partial x_j}\frac{\partial S}{\partial x_j}, E_{BB} = \frac{\partial \widetilde{\nu}_t}{\partial x_j}\frac{\partial \widetilde{\nu}_t}{\partial x_j}, C_3 = 7.0 \tag{13}$$

The turbulent eddy viscosity is defined as

$$\nu_t = f_\mu \tilde{\nu}_t, f_\mu = \frac{\chi^3}{\chi^3 + C_w^3}, \chi = \frac{\tilde{\nu}_t}{\nu}, C_w = 9.1 \quad (14)$$

The value of $\tilde{\nu}_t$ at smooth and viscous solid walls is prescribed to be zero while the value of $\tilde{\nu}_t$ for the freestream is set to be 3ν to 5ν.

3. Numerical methods

A new one-equation turbulence model is developed based on the open-source CFD toolbox, OpenFOAM [14], and is discretized using the Gaussian integration scheme based on the finite-volume method (FVM). The OpenFOAM solver used in the present study is the simpleFoam, which utilizes the Semi-Implicit Method for Pressure Linked Equations (SIMPLE) algorithm for pressure-velocity coupling. The central difference scheme (i.e., Gauss linear) is used for all gradient terms while a second-order upwind difference scheme (i.e., Gauss linearUpwind) is used to discretize the convection terms in all equations. The central difference interpolation scheme (i.e., linear) is used while the Gauss linear corrected is used for the Laplacian term. An explicit non-orthogonal correction method (i.e., corrected) is used for surface-normal gradients.

4. Results and discussion

Several typical benchmark flow cases are used based on an open source CFD software OpenFOAM to evaluate the performance of the newly proposed and developed one-equation turbulence model based on the two-equation SKE and WKO turbulence models. Meshes in all typical benchmark flow cases are obtained from the website of the Langley Research Center Turbulence Modeling Resource (TMR) [15]. Mesh independence study is performed for all cases where the value of the maximum wall y^+ is less than 1. Except for the third-order velocity gradient term, the one-equation Wray-Agarwal turbulence model (WA) [7] and the two-equation SST *k-ω* turbulence model (SST *k-ω*) [13] are very similar with the newly developed one-equation turbulence model. Thus, the numerical results of the new one-equation turbulence model are fully validated and compared with the WA and SST *k-ω* turbulence models [7, 13], the high-accuracy NASA codes (i.e., CFL3D and FUN3D) [15] and the experimental results.

4.1. Flow over a flat plate at zero pressure gradient

The classical case for a turbulence modelling testing and validation is the flat plate with zero pressure gradient provided by the NASA TMR [15]. The flow configuration is shown in Figure 1(a) with the initial boundary conditions [15]. Two meters long of solid wall and one-third meter of symmetry boundary conditions are prescribed to obtain a uniform inlet flow. A far field Riemann boundary condition (BC) is also prescribed. The static pressure at the outlet, P is equal to the reference pressure, P_{ref}, while the total pressure at the inlet, P_t is $1.02828P_{ref}$. The Reynolds number, $Re_x = \rho U_{ref} x/\mu$, based on the distance, x from the leading edge of a flat plate where U_{ref} is the uniform inlet velocity, and ρ and μ are the fluid density and dynamic viscosity, respectively. The Mach, Ma and Reynolds, Re_x numbers at the inlet are 0.2 and 5×10^6 based on x= 1 m.

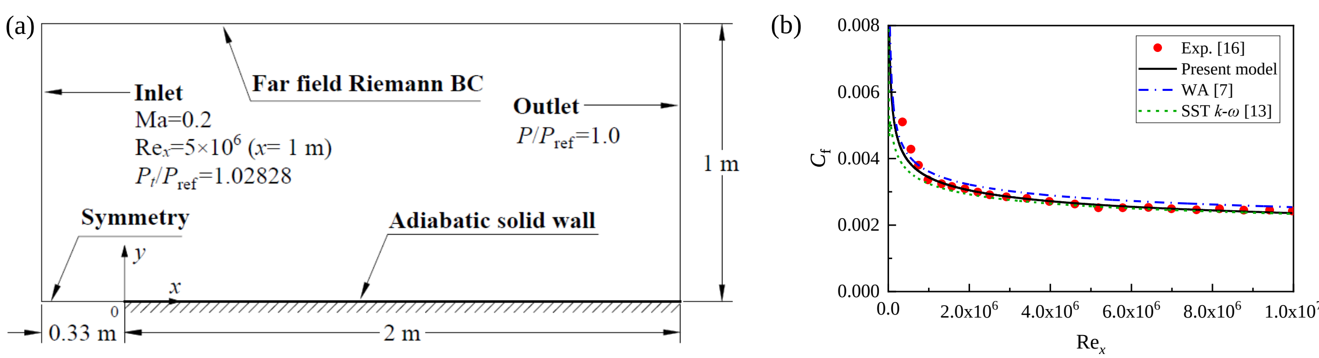


Fig. 1: (a) Configuration of the turbulent flow over a flat plate; (b) Wall skin friction coefficients for different Reynolds numbers.

The quantity of interest is the wall skin friction coefficient, $C_f = \tau_w/(\rho U_{ref}^2/2)$ in the x direction where τ_w is the skin shear stress on a surface. The numerical results of C_f is compared with the experimental results [16] as well as the selected one-equation WA and two-equation SST k-ω turbulence models, along the streamwise Re_x as shown in Figure 1(b). The results show that the one-equation WA turbulence model [7] has the best prediction of C_f at $Re_x < 1.0\times10^6$, but overpredicts C_f at the rest of Re_x region. However, the values of C_f of the present new one-equation turbulence model agree well with that of the experimental results. The new one-equation turbulence model performs much better in predicting C_f along the flat plate than the two-equation SST k-ω turbulence model [13].

4.2. Bump-in-channel flow

The main difference between the turbulent flat plate flow and bump-in-channel flow is that the latter involves pressure gradients due to the wall curvature. The flow configurations in terms of the bump in a channel and the initial boundary conditions [15] are not shown here due to page limitation. The solid wall including the bump extends from x= 0 to 1.5 m, and the symmetry boundary conditions with 25 m long are prescribed to the upstream and downstream of the solid bump wall. The Ma and Re_l based on the inlet velocity and length l= 1 m are 0.2 and 3×10^6, respectively. The static pressure at the outlet, P is equal to the reference pressure, P_{ref}, while the total pressure at the inlet, P_t is $1.02828P_{ref}$.

Figure 2(a) shows the wall skin friction coefficient, C_f along the bump wall when compared with those numerical data obtained from CFL3D and FUN3D codes [15]. The SST k-ω turbulence model [13] has the best prediction of C_f at x< 0.15 m, but underpredicts the wall skin friction coefficient at $0.6 < x < 0.8$ m. Although the WA turbulence model [7] predicts C_f well at $0.6 < x < 0.8$ m, it overpredicts the values at the rest of x region. The present new one-equation turbulence model performs better in predicting C_f than the WA turbulence model at x< 0.15 m and has the better prediction of C_f than other turbulence models at x> 0.15 m. The pressure coefficients, $C_p = (P_w - P_{ref})/(\rho U_{ref}^2/2)$ where P_w is the wall static pressure, along the bump wall simulated by all these turbulence models have an excellent agreement with the results of CFL3D and FUN3D codes [15] as shown in Figure 2(b).

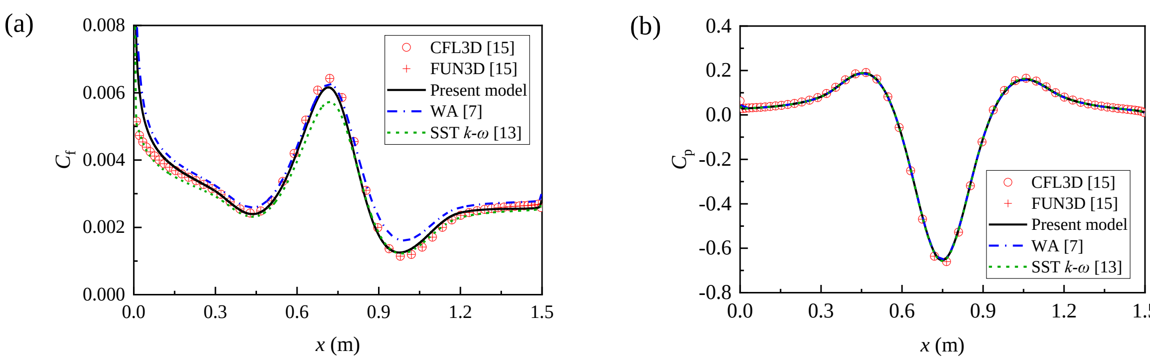


Fig. 2: (a) Wall skin friction coefficient along the bump wall; (b) Pressure coefficient along the bump wall.

4.3. Backward facing step flow

The backward facing step flow is used as the benchmark validation, which is widely used for evaluating the turbulence models, due to its complicated flow mechanism but it is a simple geometric configuration. The backward facing step geometry and flow initial boundary conditions [17] are not shown here due to page limitation. The step height, H is 12.7 mm, and the height of the inlet channel before the step is $8H$. The inlet channel is $110H$ long before the step to ensure a fully developed turbulent flow condition in the numerical simulation. The distance between the step and outlet is $50H$ which is far larger than the distance from the flow separation point to the reattachment point of the flow. The Ma and Re_H based on the inlet velocity and step height are 0.128 and 3.6×10^4, respectively.

Figure 3(a) shows the wall skin friction coefficient, C_f along the step wall when compared with experimental data [17]. The present new one-equation turbulence model has the better prediction of C_f before the step than other turbulence models. When the flow is separated, the new one-equation turbulence model and one-equation WA turbulence model [7] predict precisely the values of C_f while the two-equation SST k-ω turbulence model [13] substantially overpredicts the values of C_f.

In the reattachment region, the two-equation SST k-ω turbulence model underestimates the values of C_f, and the experimental results best match the numerical results of the one-equation WA turbulence model. The numerical results of the new one-equation turbulence model have an acceptable agreement with the experimental data [17]. But it is worth noting that the new one-equation turbulence model provides better prediction of the wall skin friction coefficient recovery after the reattachment point, while the WA and the SST k-ω turbulence models slowly recover the wall skin friction coefficients after the flow separation. The main reason is that the effect of the third-order velocity gradients can increase the turbulent diffusion which is also found in the research study of Elkhoury [1]. The pressure coefficient, C_p along the step wall is shown in Figure 3(b). Compared with the experimental data [17], there are the overpredictions of C_p by all these turbulence models at x/H< 3, while they have a good agreement with the experimental results at x/H> 3. Of all these turbulence models, the new one-equation turbulence model performs comparably with the two-equation SST k-ω turbulence model [13] and has better prediction of C_p than the one-equation WA turbulence model [7].

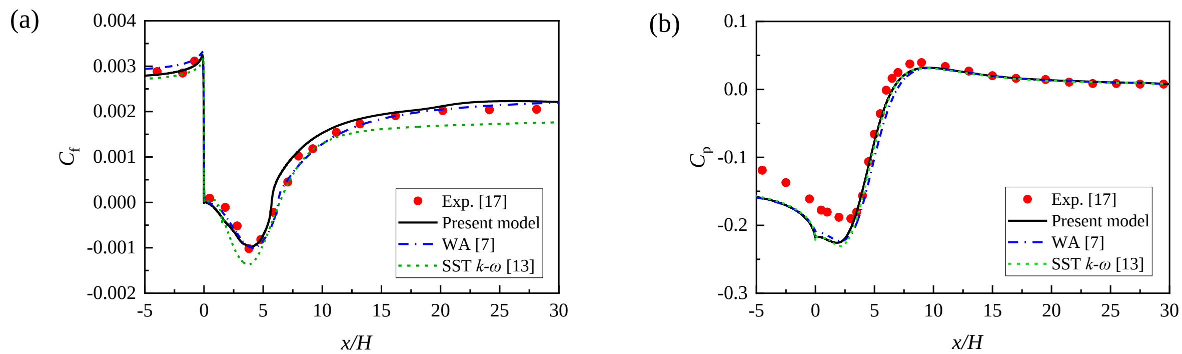
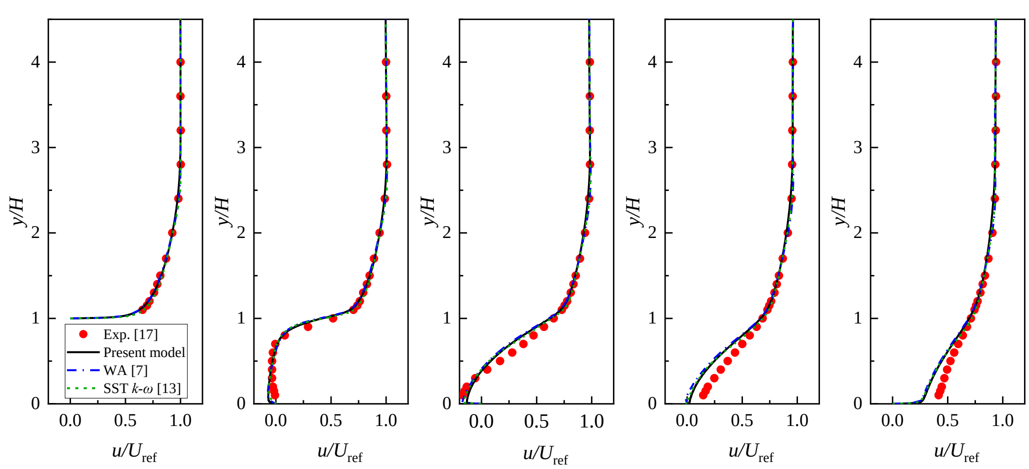


Fig. 3: (a) Wall skin friction coefficient along the step wall; (b) Pressure coefficient along the step wall.

The comparison of the velocity profiles at various x/H locations with the experimental results [17] is shown in Figure 4. The velocity profiles obtained by all these turbulence models are very close, and agree well with the experimental data including the present new one-equation turbulence model. It implies that the introduction of the third-order velocity derivative term in the new one-equation turbulence model has little effect on the velocity profile near the boundary layer, which is also found in the research study of Elkhoury [1].



(a) x/H= -4 (b) x/H= 1 (c) x/H= 4 (d) x/H= 6 (e) x/H= 10

Fig. 4: Velocity profiles of the backward facing step at x/H= -4, 1, 4, 6 and 10.

4.4. NASA wall-mounted hump separated flow

The NASA wall-mounted hump separated flow configuration is also used as the benchmark validation. The accuracy of this benchmark test case is very challenging for many turbulence models. The configuration of wall-mounted hump separated flow [18] and initial boundary conditions [19] are not shown here due to page limitation. The hump chord length, c is 420 mm and the height of the inlet channel before the start of the hump is 2.17c. The inlet channel has a length of 15.2c before the start of the hump and the distance from the end of the hump to the outlet is 7.15c. The Ma and Re_c based on the inlet velocity and hump chord are 0.1 and 9.36×10^5, respectively. P_{ref} is the reference pressure, and the static pressure at the outlet, $P= 0.99962P_{ref}$, while the total pressure at the inlet, P_t is $1.007P_{ref}$.

Figure 5(a) shows the wall skin friction coefficient, C_f along the hump wall when compared with the experimental data [19]. Before the flow separation point, the values of C_f of the present new one-equation turbulence model along the hump wall agree well with that of the experimental results. The new one-equation turbulence model also provides better prediction of C_f than the one-equation WA turbulence model [7] in the flow separation region. Compared with the experimental data, the new one-equation turbulence model has a closer prediction of the reattachment point and the wall skin friction coefficient than that of other turbulence models after the reattachment point. The new one-equation turbulence model provides better prediction of the wall skin friction coefficient recovery after the reattachment point as shown in previous test case in Section 4.3. Figure 5(b) shows the pressure coefficient, C_p along the hump wall. In most regions, the new one-equation turbulence model predicts the pressure coefficients along the hump wall better than the WA turbulence model [7] when compared with the experimental data [19], and also predicts better in the recovery of C_p after the flow reattachment.

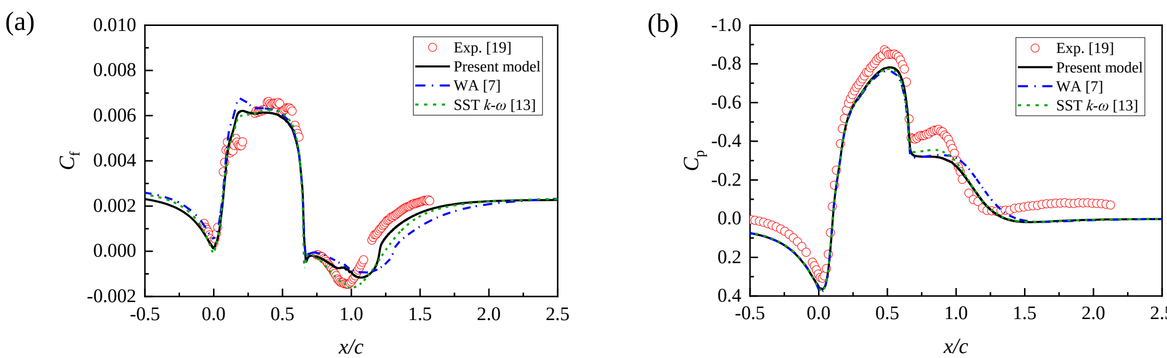


Fig. 5: (a) Wall skin friction coefficient along the hump wall; (b) Pressure coefficient along the hump wall.

5. Conclusion

A new one-equation turbulence model based on the two-equation standard k-ε and Wilcox's k-ω turbulence models is proposed and developed for validating the benchmark flow configurations including the flow over a flat plate at zero pressure gradient, the bump-in-channel flow, the backward facing step flow and the NASA wall-mounted hump separated flow. The numerical results of the wall skin friction and pressure coefficients of the new one-equation turbulence model are fully validated and compared with the results of the experimental dataset, the one- and two-equation turbulence models and the high-accuracy NASA codes (i.e., CFL3D and FUN3D). The new one-equation turbulence model makes better prediction on the turbulent flow over a flat plate than one-equation Wray-Agarwal (WA) and two-equation shear stress transport (SST) k-ω turbulence models. In addition, the new one-equation turbulence model almost always outperforms than the two-equation SST k-ω turbulence model for all benchmark test cases. It has also a better performance in simulating bump-in-channel flow and NASA wall-mounted hump separated flow than the one-equation WA turbulence model. The main feature of the new one-equation turbulence model has the improvement capacity in the prediction of the recovery of pressure and wall skin friction coefficients after reattachment point without affecting on the boundary-layer velocity profiles. It demonstrates that the present new one-equation turbulence model has a great potential to predict turbulent flow separation and reattachment.

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