

# Two-Phase Flow in an Unconsolidated Porous Medium: A Theoretical Derivation of a Macroscopic Model

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**Abstract** – In this paper, the volume averaging method is applied to an unconsolidated granular medium with a two-phase boiling flow and an internal heat source in the solid. The main steps of the theoretical derivation of the macroscopic averaged conservation equations and their closure problems are given. The main contributions of this work concern the mobile character of the solid phase and the vapour flow rate due to boiling. A closure for the vapour flow rate is also proposed.

**Keywords:** Severe Accident, SFR, Debris Bed, Three-Phase Flow, Granular Flow, Porous Media, Volume Averaging

## 1. Introduction

The French strategy to mitigate severe accidents in a Sodium cooled Fast neutron Reactor (SFR) relies on relocating the molten core material (corium) below the core region as quickly as possible, where a dedicated safety device, the core catcher, collects them to protect the primary vessel [1]. It is widely accepted [2] [3] that the incoming liquid corium would be fragmented into solid particles when it arrives in the lower plenum due to violent interactions with the coolant (liquid sodium). When these fragments settle, they form a so-called debris bed [3], which essentially contains fuel and steel particles. The decay heat of the fission products, volumetrically released in the fuel particles, must be evacuated from the medium in order to ensure its cooling and geometrical stabilization, i.e. to avoid re-melting of the debris and to protect the core-catcher and the primary vessel. Furthermore, it is important to evaluate whether the debris bed can reach a configuration that induces a positive neutron reactivity, and if so, what can be done to avoid this. For all these reasons, it is important to quantitatively study the behaviour of the debris bed, which implies the description of the following phenomena: heat transfer between particles and sodium, sodium vaporization and debris transport, leading to the so-called “self-levelling” effect [4].

A convenient way to reach this objective consists in considering the debris bed as an unconsolidated granular medium with an internal heat source exchanging with an intern boiling flow of sodium. The intrinsic complexity of this problem requires homogenized equations obtained by use of an up-scaling method. Among the existing methods, the volume averaging has been successfully applied to the description of single-phase viscous and inertial flows [5], multi-phase viscous [6] and inertial [7] flows, heat and mass transfers [8] in fixed porous media, and dense particle flows in single phase viscous flows [9]. However, the more complex situation of particles entrainment by a multi-phase boiling flow has not received the same attention in the literature. This work constitutes a first attempt to address this problem, by investigating the possible form of the macroscopic equations and the required closure relations. The theoretical derivation of the macroscopic conservation equations is presented, taking into account the particularity of the mobility of the solid phase (particles) and the transfers between the liquid and the gas phases (boiling).

## 2. Theoretical Background: Volume Averaging Method

This paper uses the averaging operators as defined in [7] [8] [9]. The Volume Averaging method consists in applying the operator of the Eq. (1) to the local boundary value problem to obtain the unclosed macroscopic equations. The most important relations are the theorems relative to the average of the temporal and spatial derivatives Eqs. (2) and (3) and the similar one for vector quantities [10]. In a second step the local and averaged scales are separated by introducing the so-

called Gray decomposition [11] Eq. (4). Closure problems, at the scale of the deviation, are derived to close the macroscopic equations.

$$\langle \psi_k \rangle = \frac{1}{V} \iiint_{V_k} \psi_k dV = \varepsilon_k \langle \psi_k \rangle^k \quad (1)$$

$$\left\langle \frac{\partial \psi_k}{\partial t} \right\rangle = \frac{\partial \langle \psi_k \rangle}{\partial t} - \frac{1}{V} \iint_{A_k} \psi_k \mathbf{w}_k \cdot \mathbf{n}_k dA \quad (2)$$

$$\langle \nabla \psi_k \rangle = \nabla \langle \psi_k \rangle + \frac{1}{V} \iint_{A_k} \psi_k \mathbf{n}_k dA \quad (3)$$

$$\psi_k = \langle \psi_k \rangle^k + \widetilde{\psi}_k^k \quad (4)$$

where  $\psi_k$  is a scalar quantity of the phase  $k$ ,  $\langle \psi_k \rangle$  and  $\langle \psi_k \rangle^k$  represent respectively the superficial and intrinsic averages of  $\psi_k$  quantity,  $\widetilde{\psi}_k^k$  the deviation of the local value of  $\psi_k$  to the intrinsic average  $\langle \psi_k \rangle^k$ ,  $t$  is the time.  $V$  is the control volume. It is a Representative Elementary Volume (REV) as represented in Fig. 1.  $V_k$  and  $A_k$  are respectively the control volume and the interface area of the phase  $k$ ,  $\mathbf{n}_k$  is the normal vector oriented towards the outside of phase  $k$ .  $\varepsilon_k = \frac{V_k}{V}$  is the volume occupation rate of the phase  $k$  and  $\mathbf{w}_k$  is the interfacial velocity.

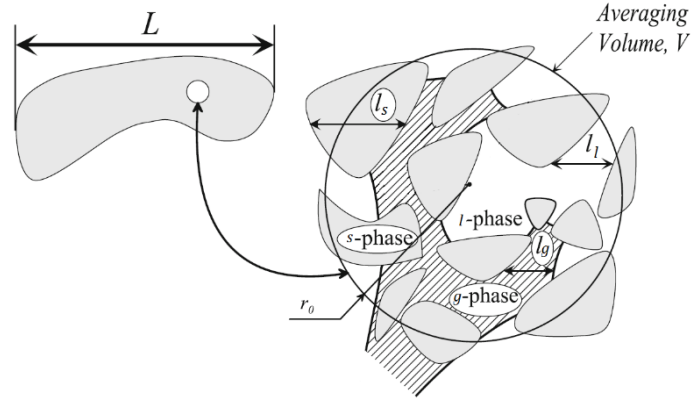


Fig. 1: Schema of the system's representative elementary volume (REV) [7]

### 3. Application to the debris bed

#### 3.1. Local boundary-value problem

The local boundary value problem represents two incompressible, immiscible, Newtonian fluids, exchanging heat and mass through their interface, to model the liquid ( $l$ ) and vapour ( $g$ ) sodium and a third incompressible, immiscible, Newtonian pseudo-fluid [9] with a volumetric heat source term representing the solid particles ( $s$ ) considering local thermal non-equilibrium. The conservation equations for mass Eq. (5), momentum Eq. (6), and energy Eq. (7), and the associated boundary conditions Eqs. (8) - (16) are the following:

$$\nabla \cdot \mathbf{v}_k = 0 \quad k = l, g, s \quad (5)$$

$$\frac{\partial \mathbf{v}_k}{\partial t} + \nabla \cdot (\mathbf{v}_k \mathbf{v}_k) = \kappa_k \mathbf{g} - \frac{1}{\rho_k} \nabla P_k + \frac{\eta_k}{\rho_k} \nabla^2 \mathbf{v}_k \quad k = l, g, s \quad (6)$$

$$c_{pk} \left( \frac{\partial \theta_k}{\partial t} + \nabla \cdot (\theta_k \mathbf{v}_k) \right) = Q_k + \frac{\lambda_k}{\rho_k} \nabla^2 \theta_k \quad k = l, g, s \quad (7)$$

$$\begin{aligned}
\rho_l(\mathbf{w}_{lg} - \mathbf{v}_l) \cdot \mathbf{n}_{lg} &= -\rho_g(\mathbf{w}_{lg} - \mathbf{v}_g) \cdot \mathbf{n}_{gl} = \varphi_{lg} && \text{on } A_{lg} && (8) \\
\mathbf{v}_l &= \mathbf{v}_g && \text{on } A_{lg} && (9) \\
\mathbf{w}_{fs} &= \mathbf{v}_f && \text{on } A_{fs} && f = l, g && (10) \\
\mathbf{w}_{fs} &= \mathbf{v}_s && \text{on } A_{fs} && f = l, g && (11) \\
\varphi_{lg}(\mathbf{v}_l - \mathbf{v}_g) + P_l \mathbf{n}_{lg} - \eta_l(\nabla \mathbf{v}_l + \nabla \mathbf{v}_l^T) \cdot \mathbf{n}_{lg} + P_g \mathbf{n}_{gl} - \eta_g(\nabla \mathbf{v}_g + \nabla \mathbf{v}_g^T) \cdot \mathbf{n}_{gl} &= \mathcal{J}_{lg} H_{lg} \mathbf{n}_{lg} && \text{on } A_{lg} && (12) \\
P_f \mathbf{n}_{fs} - \eta_f(\nabla \mathbf{v}_f + \nabla \mathbf{v}_f^T) \cdot \mathbf{n}_{fs} + P_s \mathbf{n}_{sf} - \eta_s(\nabla \mathbf{v}_s + \nabla \mathbf{v}_s^T) \cdot \mathbf{n}_{sf} &= \mathcal{J}_{fs} H_{fs} \mathbf{n}_{fs} && \text{on } A_{fs} && f = l, g && (13) \\
\theta_f &= 0 && \text{on } A_{lg} && f = l, g && (14) \\
\theta_f &= \theta_s && \text{on } A_{fs} && f = l, g && (15) \\
\lambda_f \nabla \theta_f \cdot \mathbf{n}_{fs} + \lambda_s \nabla \theta_s \cdot \mathbf{n}_{sf} &= 0 && \text{on } A_{fs} && f = l, g && (16) \\
-\Delta h_{vap} \varphi_{lg} + \lambda_l \nabla \theta_l \cdot \mathbf{n}_{lg} + \lambda_g \nabla \theta_g \cdot \mathbf{n}_{gl} &= 0 && \text{on } A_{lg} && (17)
\end{aligned}$$

where  $\mathbf{v}_k$  is the velocity of phase  $k$ ,  $\mathbf{g}$  is the gravity acceleration,  $\rho_k$  is the density,  $P_k$  is the pressure,  $\eta_k$  is the dynamic viscosity,  $C_{pk}$  is the specific heat capacity and  $\lambda_k$  is the thermal conductivity.  $\kappa_k$  is the thermal dilatation coefficient of the phase  $k$  supposed linear in relative temperature, following the so-called Boussinesq approximation [12], with  $\kappa_s = 1$  as the dilatation of the solid phase is neglected.  $Q_k$  is the specific heat source, with  $Q_l = Q_g = 0 \text{ W.kg}^{-1}$ . The energy conservation is written in terms of the difference between the phase temperature and the saturation temperature of the sodium  $T^{sat}$ ,  $\theta_k = T_k - T^{sat}$ .  $\varphi_{lg}$  is the mass flux at the gas-liquid interface due to phase change.  $A_{kk'}$  is the area of the interface between the phases  $k$  and  $k'$ . The surface tension between the phases  $k$  and  $k'$  is  $\mathcal{J}_{kk'}$  and  $H_{kk'}$  is the mean curvature of this interface.  $\Delta h_{vap}$  is the latent heat of vaporisation of the sodium.

### 3.2. Non-closed averaged equations & Deviation problem

Following the Volume Averaging method, similarly to the proposition of Lasseux et al. [7] and Duval et al. [8], the volume average operator (Eq. (1)) is applied to the precedent local boundary-value problem (Eqs. (5), (6) and (7)). Then the Gray decomposition (Eq. (4)) is introduced to derive the non-closed averaged conservation equations (Eqs. (18), (19) and (20) hereafter). It is written under the assumptions that physical properties are uniform on the REV, that phase change occurs uniformly on all the liquid-gas interface, and that conditions of Eqs. (21) and (22) are verified.

$$\frac{\partial \varepsilon_k}{\partial t} + \nabla \cdot (\varepsilon_k \langle \mathbf{v}_k \rangle^k) = \frac{1}{V} \iint_{A_k} (\mathbf{w}_{kk'} - \mathbf{v}_k) \cdot \mathbf{n}_{kk'} dA = (\delta_{kl} - \delta_{kg}) \frac{1}{\rho_k} \dot{m}_{lg} \quad k = l, g, s \quad (18)$$

$$\begin{aligned}
\varepsilon_k \frac{\partial \langle \mathbf{v}_k \rangle^k}{\partial t} + (\varepsilon_k \langle \mathbf{v}_k \rangle^k \cdot \nabla) \langle \mathbf{v}_k \rangle^k \\
= \frac{1}{V} \iint_{A_k} \widetilde{\mathbf{v}}_k^k (\mathbf{w}_{kk'} - \mathbf{v}_k) \cdot \mathbf{n}_{kk'} dA + \varepsilon_k \langle \kappa_k \rangle^k \mathbf{g} - \frac{\varepsilon_k}{\rho_k} \nabla \langle P_k \rangle^k + \underbrace{\frac{\eta_k}{\rho_k} \nabla^2 (\varepsilon_k \langle \mathbf{v}_k \rangle^k) - \frac{\eta_k}{\rho_k} \nabla \langle \mathbf{v}_k \rangle^k \cdot \nabla \varepsilon_k}_{\text{Brinkman's terms}} \quad k = l, g, s \quad (19)
\end{aligned}$$

$$\begin{aligned}
+ \frac{1}{V} \iint_{A_k} \left( -\frac{1}{\rho_k} \widetilde{P}_k^k \mathbf{n}_{kk'} + \frac{\eta_k}{\rho_k} (\nabla \widetilde{\mathbf{v}}_k^k) \cdot \mathbf{n}_{kk'} \right) dA \\
C_{pk} \left( \varepsilon_k \frac{\partial \langle \theta_k \rangle^k}{\partial t} + \varepsilon_k \langle \mathbf{v}_k \rangle^k \cdot \nabla \langle \theta_k \rangle^k \right) \\
= -(\delta_{kl} - \delta_{kg}) \frac{C_{pk} \langle \theta_k \rangle^k}{\rho_k} \dot{m}_{lg} + \varepsilon_k Q_k + \frac{\lambda_k}{\rho_k} \nabla^2 (\varepsilon_k \langle \theta_k \rangle^k) - \frac{\lambda_k}{\rho_k} \nabla \varepsilon_k \cdot \nabla \langle \theta_k \rangle^k + \frac{1}{V} \iint_{A_k} \frac{\lambda_k}{\rho_k} \nabla \widetilde{\theta}_k^k \cdot \mathbf{n}_{kk'} dA \quad k = l, g, s \quad (20)
\end{aligned}$$

$$\frac{\rho_k \|\widehat{\mathbf{v}}_k^k\| l_k}{\eta_k L \varepsilon_k} \ll 1 \quad k = l, g, s \quad (21)$$

$$\frac{C_{pk} \rho_k \|\widehat{\mathbf{v}}_k^k\| l_k}{\lambda_k L \varepsilon_k} \ll 1 \quad k = l, g, s \quad (22)$$

where  $\delta_{kk'}$  is the Kronecker symbol,  $\dot{m}_{lg} = \frac{1}{V} \iint_{A_{lg}} \varphi_{lg} dA$  is the volumetric boiling rate,  $A_k = \cup_{k' \neq k} A_{kk'}$  with  $A_{kk'}$  the interfacial area between the phase  $k$  and  $k'$ .  $\mathbf{w}_{kk'} = -\mathbf{w}_{k'k}$  is the velocity of the interface between the phase  $k$  and  $k'$  and  $\mathbf{n}_{kk'}$  the normal vector to this interface oriented outside of the phase  $k$ .  $l_k$  is the characteristic pore-scale length and  $L$  is the characteristic length of the system.  $\|\cdot\|$  is the vector norm.

There are some differences in these conservation equations compared to precedent works [7] [8]. As the phase change between the liquid and gas phases is considered,  $(\mathbf{w}_{kk'} - \mathbf{v}_k)$  on  $A_{lg}$  is not zero. The Brinkman's terms in the momentum balance equation (Eq. (19)) and an analogous one in the energy balance equation (Eq. (20)), often considered as negligible inside the medium, are kept because they may become significant at the boundary of the medium (i.e. at the surface of the heap), which is a zone of interest to simulate the self-levelling.

Eqs. (18), (19) and (20) depend on the deviation quantities, so the scales are not yet separated, and the averaged problem is unclosed. Alongside the averaged equations, the difference between the averaged equations and the local boundary value problem results in the deviation problem. Eqs. (23), (24) and (25) figure the conservation equations in this problem, under the following assumption: i) the terms relative to the mass exchange by sodium boiling, too complex, are neglected at the scale of the deviation problem; and ii) the effect of the thermal dilatation is neglected at the local scale. Those hypothesis are generally considered in the literature [8].

$$\nabla \cdot \widehat{\mathbf{v}}_k^k = 0 \quad k = l, g, s \quad (23)$$

$$\varepsilon_k \mathbf{v}_k \cdot \nabla \cdot \widehat{\mathbf{v}}_k^k = -\frac{\varepsilon_k}{\rho_k} \nabla \widehat{P}_k^k + \varepsilon_k \frac{\eta_k}{\rho_k} \nabla^2 \widehat{\mathbf{v}}_k^k - \frac{1}{V} \iint_{A_k} \left( -\frac{1}{\rho_k} \widehat{P}_k^k \mathbf{n}_{kk'} + \frac{\eta_k}{\rho_k} (\nabla \widehat{\mathbf{v}}_k^k) \cdot \mathbf{n}_{kk'} \right) dA \quad k = l, g, s \quad (24)$$

$$C_{pk} \left( \varepsilon_k \mathbf{v}_k \cdot \nabla \widehat{\theta}_k^k + \varepsilon_k \widehat{\mathbf{v}}_k^k \cdot \nabla (\theta_k)^k \right) = \frac{\lambda_k}{\rho_k} \varepsilon_k \nabla^2 \widehat{\theta}_k^k - \frac{1}{V} \iint_{A_k} \frac{\lambda_k}{\rho_k} \nabla \widehat{\theta}_k^k \cdot \mathbf{n}_{kk} dA \quad k = l, g, s \quad (25)$$

According to work of Lasseux et al. [7], and within the conditions represented by Eqs. (26) and (27), the averaged pressures and the superficial tension compensate each other. Furthermore, if the inequality Eq. (28) is verified, then the viscous dissipation due to the averaged part of the velocities at the interface is negligible.

$$\frac{\eta_k \langle \|\nabla \widehat{\mathbf{v}}_k^k\| \rangle_{kk'}}{\mathcal{T}_{kk'} \langle H_{kk'} \rangle_{kk'}} = Ca_k \frac{\langle \|\nabla \widehat{\mathbf{v}}_k^k\| \rangle_{kk'}}{\langle H_{kk'} \rangle_{kk'} \|\langle \mathbf{v}_k \rangle^k\|} \ll 1 \quad k = l, g, s \quad (26)$$

$$\frac{\rho_k l_k \langle \|\mathbf{v}_k \cdot \nabla \widehat{\mathbf{v}}_k^k\| \rangle_{kk'}}{\mathcal{T}_{kk'} \langle H_{kk'} \rangle_{kk'}} = We_k \frac{\langle \|\mathbf{v}_k \cdot \nabla \widehat{\mathbf{v}}_k^k\| \rangle_{kk'}}{\langle H_{kk'} \rangle_{kk'} \|\langle \mathbf{v}_k \rangle^k\|^2} \ll 1 \quad k = l, g, s \quad (27)$$

$$\|\langle \mathbf{v}_k \rangle^k\| \frac{l_k}{L} \ll \|\widehat{\mathbf{v}}_k^k\| \quad k = l, g, s \quad (28)$$

where  $\langle \psi_k \rangle_{kk'}$  refers to the surface averaging on the interface area  $A_{kk'}$  of  $\psi_k$ .  $Ca_k$  is the capillary number considering the average velocity and  $We_k$  is the Weber number based on the average velocity for the phase  $k$ .

Under these conditions the boundary conditions associated to the deviation problem are:

$$\widetilde{\mathbf{v}}_k^k = \widetilde{\mathbf{v}}_{k'}^{k'} - (\langle \mathbf{v}_k \rangle^k - \langle \mathbf{v}_{k'} \rangle^{k'}) \quad \text{on } A_{kk'} \quad \begin{matrix} k, k' = l, g, s \\ k \neq k' \end{matrix} \quad (29)$$

$$(\widetilde{P}_k^k - \widetilde{P}_{k'}^{k'}) \mathbf{n}_{kk'} - \eta_k (\nabla \widetilde{\mathbf{v}}_k^k + \nabla \widetilde{\mathbf{v}}_k^{kT}) \cdot \mathbf{n}_{kk'} - \eta_{k'} (\nabla \widetilde{\mathbf{v}}_{k'}^{k'} + \nabla \widetilde{\mathbf{v}}_{k'}^{k'T}) \cdot \mathbf{n}_{k'k} = 0 \quad \text{on } A_{kk'} \quad \begin{matrix} k, k' = l, g, s \\ k \neq k' \end{matrix} \quad (30)$$

$$\widetilde{\theta}_f^f = -\langle \theta_f \rangle^f \quad \text{on } A_{lg} \quad f = l, g \quad (31)$$

$$\widetilde{\theta}_f^f = \widetilde{\theta}_s^s - (\langle \theta_f \rangle^f - \langle \theta_s \rangle^s) \quad \text{on } A_{fs} \quad f = l, g \quad (32)$$

$$(\lambda_f \nabla \widetilde{\theta}_f^f - \lambda_s \nabla \widetilde{\theta}_s^s + \lambda_f \nabla \langle \theta_f \rangle^f - \lambda_s \nabla \langle \theta_s \rangle^s) \cdot \mathbf{n}_{fs} = 0 \quad \text{on } A_{fs} \quad f = l, g \quad (33)$$

### 3.3. Closure

To finally separate the macroscopic and pore-scale effects, linear relations are assumed between the deviations and the averaged quantities which are the sources of the deviation problem. In our problem, the sources are the averaged velocities, the averaged relative temperatures and the gradients of the averaged temperatures. The linearity coefficients are called the closure variables. Under condition that dynamic and thermodynamic effects are separated [8], the closure relations are the following:

$$\widetilde{\mathbf{v}}_k^k = \sum_{j=l,g,s} \overline{\overline{a_{kj}}} \cdot \langle \mathbf{v}_j \rangle^j \quad k = l, g, s \quad (34)$$

$$\widetilde{P}_k^k = \sum_{j=l,g,s} \eta_k \mathbf{b}_{kj} \cdot \langle \mathbf{v}_j \rangle^j \quad k = l, g, s \quad (35)$$

$$\widetilde{\theta}_k^k = \sum_{j=l,g,s} c_{kj} \langle \theta_j \rangle^j + \sum_{j=l,g,s} \mathbf{d}_{kj} \cdot \nabla \langle \theta_j \rangle^j \quad k = l, g, s \quad (36)$$

where  $\overline{\overline{a_{kj}}}$  and  $\mathbf{b}_{kj}$  are respectively the closure variables for the velocity deviations and the pressure deviations of phase  $k$  associated to the averaged velocity of the phase  $j$ .  $c_{kj}$  and  $\mathbf{d}_{kj}$  are the closure variables for the deviation of the relative temperatures of the phase  $k$ , respectively associated to the averaged relative temperature of the phase  $j$  and their gradient.

The closure variables may be determined by resolving the closure problems. One closure problem per source is written by injecting closure relations Eqs. (34), (35) and (36) into the deviation problem Eqs. (23), (24), (25) and their boundary conditions Eqs. (29) - (33).

The closure problem associated to the averaged velocity of the phase  $j$  is the following:

$$\nabla \cdot \overline{\overline{a_{kj}}} = 0 \quad k = l, g, s \quad (37)$$

$$\frac{\rho_k}{\eta_k} \mathbf{v}_k \cdot \nabla \overline{\overline{a_{kj}}} = -\nabla \mathbf{b}_{kj} + \nabla^2 \overline{\overline{a_{kj}}} - \frac{1}{V_k} \iint_{A_k} \mathbf{n}_{kk'} \cdot (-\overline{\overline{\mathbf{I} \mathbf{b}_{kj}}} + \nabla \overline{\overline{a_{kj}}}) dA \quad k = l, g, s \quad (38)$$

$$\overline{\overline{a_{kj}}} = \overline{\overline{a_{k'j}}} - (\delta_{jk} - \delta_{jk'}) \overline{\overline{\mathbf{I}}} \quad \text{on } A_{kk'} \quad \begin{matrix} k, k' = l, g, s \\ k \neq k' \end{matrix} \quad (39)$$

$$\mathbf{n}_{kk'} \cdot \left[ \eta_k \left( \overline{\overline{\mathbf{I} \mathbf{b}_{kj}}} - (\nabla \overline{\overline{a_{kj}}} + \nabla \overline{\overline{a_{kj}}^T}) \right) - \eta_{k'} \left( \overline{\overline{\mathbf{I} \mathbf{b}_{k'j}}} - (\nabla \overline{\overline{a_{k'j}}} + \nabla \overline{\overline{a_{k'j}}^T}) \right) \right] = 0 \quad \text{on } A_{kk'} \quad \begin{matrix} k, k' = l, g, s \\ k \neq k' \end{matrix} \quad (40)$$

The closure problem associated to the averaged relative temperature of the phase  $j$  is the following:

$$\frac{\rho_k C_{pk}}{\lambda_k} \mathbf{v}_k \cdot \nabla c_{kj} = \nabla^2 c_{kj} - \frac{1}{V_k} \iint_{A_k} \nabla c_{kj} \cdot \mathbf{n}_{kk} dA \quad k = l, g, s \quad (41)$$

$$c_{fj} = -\delta_{jf} \quad \text{on } A_{lg} \quad f = l, g \quad (42)$$

$$c_{fj} = c_{sj} - (\delta_{jf} - \delta_{js}) \quad \text{on } A_{fs} \quad f = l, g \quad (43)$$

$$(\lambda_f \nabla c_{fj} - \lambda_s \nabla c_{sj}) \cdot \mathbf{n}_{fs} = 0 \quad \text{on } A_{fs} \quad f = l, g \quad (44)$$

The closure problem associated to the gradient of the averaged temperature of the phase  $j$  is the following:

$$\frac{\rho_k C_{pk}}{\lambda_k} (\mathbf{v}_k \cdot \nabla \mathbf{d}_{kj} + \delta_{jk} \widetilde{\mathbf{v}}_k^k) = \nabla^2 \mathbf{d}_{kj} - \frac{1}{V_k} \iint_{A_k} \mathbf{n}_{kk'} \cdot \nabla \mathbf{d}_{kj} dA \quad k = l, g, s \quad (45)$$

$$\mathbf{d}_{fj} = 0 \quad \text{on } A_{lg} \quad f = l, g \quad (46)$$

$$\mathbf{d}_{fj} = \mathbf{d}_{sj} \quad \text{on } A_{fs} \quad f = l, g \quad (47)$$

$$\lambda_f (\delta_{jf} + \nabla \mathbf{d}_{fj}) = \lambda_s (\delta_{js} + \nabla \mathbf{d}_{sj}) \quad \text{on } A_{fs} \quad f = l, g \quad (48)$$

Finally, injecting the closure relations Eqs. (34), (35) and (36) into the unclosed averaged conservation equations Eqs. (18), (19) and (20) result in the macroscopic conservation equations:

$$\frac{\partial \varepsilon_k}{\partial t} + \nabla \cdot (\varepsilon_k \langle \mathbf{v}_k \rangle^k) = (\delta_{kl} - \delta_{kg}) \frac{1}{\rho_k} \dot{m}_{lg} \quad k = l, g, s \quad (49)$$

$$\begin{aligned} & \varepsilon_k \frac{\partial \langle \mathbf{v}_k \rangle^k}{\partial t} + (\varepsilon_k \langle \mathbf{v}_k \rangle^k \cdot \nabla) \langle \mathbf{v}_k \rangle^k \\ &= \varepsilon_k \langle \kappa_k \rangle^k \mathbf{g} - \frac{\varepsilon_k}{\rho_k} \nabla \langle P_k \rangle^k + \frac{\eta_k}{\rho_k} \nabla^2 (\varepsilon_k \langle \mathbf{v}_k \rangle^k) - \frac{\eta_k}{\rho_k} \nabla \langle \mathbf{v}_k \rangle^k \cdot \nabla \varepsilon_k + (\delta_{kl} + \delta_{kg}) \sum_{j=l,g,s} \langle \mathbf{v}_j \rangle^j \cdot \overline{\overline{E_{kj}^{lg}}} \quad k = l, g, s \quad (50) \end{aligned}$$

$$\begin{aligned} & + \frac{\eta_k}{\rho_k} \sum_{j=l,g,s} \langle \mathbf{v}_j \rangle^j \cdot \overline{\overline{A_{kj}}} \\ & C_{pk} \left( \varepsilon_k \frac{\partial \langle \theta_k \rangle^k}{\partial t} + \varepsilon_k \langle \mathbf{v}_k \rangle^k \cdot \nabla \langle \theta_k \rangle^k \right) \\ &= -(\delta_{kl} - \delta_{kg}) \frac{C_{pk} \langle \theta_k \rangle^k}{\rho_k} \dot{m}_{lg} + \varepsilon_k Q_k + \frac{\lambda_k}{\rho_k} \nabla^2 (\varepsilon_k \langle \theta_k \rangle^k) - \frac{\lambda_k}{\rho_k} \nabla \varepsilon_k \cdot \nabla \langle \theta_k \rangle^k \quad k = l, g, s \quad (51) \\ & + \frac{\lambda_k}{\rho_k} \sum_{j=l,g,s} \langle \theta_j \rangle^j C_{kj} + \frac{\lambda_k}{\rho_k} \sum_{j=l,g,s} \nabla \langle \theta_j \rangle^j \cdot \mathbf{D}_{kj} \end{aligned}$$

where  $\overline{\overline{A_{kj}}}$ ,  $\overline{\overline{E_{kj}^{lg}}}$ ,  $C_{kj}$  and  $\mathbf{D}_{kj}$  are the macroscopic closures for the phase  $k$  associated to the phase  $j$ , which are the effective properties of the medium. For a system with 3 phases  $l$ ,  $g$  and  $s$ , each “family” of terms includes 9 terms ( $3 \times 3$ ), resulting in 36 macroscopic closures, that may be determined by applying the following expressions to the solutions of the previously presented closure problems:

$$\overline{\overline{A}}_{kj} = \iint_{A_k} \mathbf{n}_{kk'} \cdot (-\overline{\mathbf{I}}\mathbf{b}_{kj} + \nabla \overline{\overline{a}}_{kj}) dA \quad k, j = l, g, s \quad (52)$$

$$\overline{\overline{E}}_{kj}^{lg} = \iint_{A_k} ((\mathbf{w}_{kk'} - \mathbf{v}_k) \cdot \mathbf{n}_{kk'}) \overline{\overline{a}}_{kj} dA \quad k, j = l, g, s \quad (53)$$

$$C_{kj} = \frac{1}{V} \iint_{A_k} \nabla c_{kj} \cdot \mathbf{n}_{kk'} dA \quad k, j = l, g, s \quad (54)$$

$$\mathbf{D}_{kj} = \frac{1}{V} \iint_{A_k} \mathbf{n}_{kk'} \cdot \nabla \mathbf{d}_{kj} dA \quad k, j = l, g, s \quad (55)$$

To complete this problem, in addition to the boundary conditions, the expression of the boiling rate  $\dot{m}_{lg}$  as a function of the medium's characteristics is derived, according to Duval *et al.* [8], by averaging Eq. (17) on the interfacial area  $A_{lg}$ , written under Gray decomposition Eq. (4) and by injecting the closure relations on temperature Eq. (36):

$$\dot{m}_{lg} = \frac{1}{\Delta h_{vap}} \left( \sum_{j=l,g,s} \langle \theta_j \rangle^j F_j^{lg} + \sum_{j=l,g,s} \nabla \langle \theta_j \rangle^j \cdot \mathbf{G}_j^{lg} \right) \quad (56)$$

$$F_j^{lg} = \frac{1}{V} \iint_{A_{lg}} (\lambda_l \nabla c_{lj} - \lambda_g \nabla c_{gj}) \cdot \mathbf{n}_{lg} dA \quad j = l, g, s \quad (57)$$

$$\mathbf{G}_j^{lg} = \frac{1}{V} \iint_{A_{lg}} \mathbf{n}_{lg} \cdot (\lambda_l \nabla \mathbf{d}_{lj} - \lambda_g \nabla \mathbf{d}_{gj}) + \lambda_j (\delta_{jl} - \delta_{jg}) \mathbf{n}_{lg} dA \quad j = l, g, s \quad (58)$$

where  $F_j^{lg}$ ,  $\mathbf{G}_j^{lg}$  are the 6 macroscopic closures for the volume boiling rate  $\dot{m}_{lg}$  respectively associated to the averaged relative temperature and their gradient, of the phase  $j$ .

### 3.4. Perspectives

The ultimate objective of this work is to characterize the macroscopic behaviour of the medium, i.e., to resolve the macroscopic boundary value problem given by Eqs. (49), (50) and (51). This requires appropriate values for the macroscopic properties, which may be obtained in two ways. The first one consists in resolving numerically the closure problems represented by Eqs. (37) to (48), and the second one consists in implementing ad hoc closure relations, determined experimentally or by extension of models for close comparable applications.

A numerical resolution of the closure problems requires specific efforts and further developments, as well as an explicit resolution of the local boundary value problem on a REV of the medium and in representative conditions in terms of power, velocity, pressure and temperature, which is an even more challenging task. This is clearly beyond the scope of this work, which is a first approach of the problem and ambitions to identify the most important effects before a more precise characterisation.

The second way then appears preferable, and the relevant constitutive laws will be identified in the literature, as well as the ones which are missing and thus require further research. Then the obtained modelling will be implemented to evaluate the behavior of a debris bed under self-levelling conditions.

## 4. Conclusions

In this work, the Volume Averaging method has been applied to an unconsolidated granular medium with internal volumetric heat source and a boiling sodium flow out of local thermal equilibrium. The macroscopic sources proposed are the averaged velocity, the averaged relative temperature and the gradient of the average temperature of each phase. This leads to define 36 macroscopic closures and the closure problems which determine the effective properties of the debris bed. A closure for the volume boiling rate of the sodium and their 6 associated effective properties were proposed.

The numerical resolution of the closure problems is a very challenging task. Thus, in the future, the existing empirical closure relations will be reviewed, and the model will be implemented in a numerical platform in order to get the first evaluation of this debris bed model behaviour under self-levelling conditions [13]. Then this simulation will be compared to experimental results [14] [15].

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