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Optimization Study of Power-Law Fluids Staggered Circular Cylinders in Laminar Forced Convection

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Abstract- In this study, for power-law fluids, a two-dimensional heat transfer analysis was performed in a circular cylinder to determine the ideal distance between cylinders in equilateral triangle configurations for forced convection in free stream cross-flow. The cylinder array is in contact with a free stream of a specific temperature and velocity while occupying a set volume. The optimal cylinder-to-cylinder spacing is determined by maximizing the overall thermal conductance between all the cylinders and the free stream. The numerical study was conducted to maximize the heat transfer rate over the range of Reynolds number, $40 \leq Re \leq 200$; power-law index, $0.2 \leq n \leq 1.3$; Prandtl number, $1 \leq Pr \leq 100$; and geometries with spacing from cylinder-to cylinder, $0.5 \leq S/D \leq 2$. The governing equations have been solved for the steady state flow over the range of parameters by employing finite-element numerical scheme. The flow and thermal field by using hot cylinder arranged in triangular array is analysed by plotting the streamlines and isotherms. The thermal heat conductance increases for the shear thinning fluids as Reynolds number increases and on further increasing the Prandtl number. The relation for thermal heat conductance with Prandtl number for extreme values of Reynolds number is also shown for different values of power-law index.

Keywords: circular cylinder, power-law fluids, Prandtl number, Reynolds number, cylinder-to-cylinder spacing.

1. Introduction:

It is primarily discovered that the flow and heat transfer phenomena of both Newtonian and non-Newtonian fluids through the array of cylinders are important for the numerous industrial applications. For example, the process stream flow at the shell side is often used to mimic the flow through the tube banks of tubular heat exchangers. Additionally, the bulk of nuclear reactor fuel components utilise flow across tube banks, and the heat transfer mechanism in such a process is composed of several parallel fuel tubes or rods arranged in regular arrays. These flow patterns are also demonstrated to be important in biological systems, the drying of fibrous materials in fluidized beds, and the filtering of suspensions of paper and pulp [1-3]. The local and global behaviours of the thermal and flow properties, including isotherm profiles, local and average Nusselt numbers, streamlines are the main factors of interest in each of these scenarios. Less research has been done over the years to estimate these parameters for the incompressible Newtonian fluids over the periodic array of cylinders and/or over tube banks [4-6]. It is also pertinent to add that even with the most prevalent Newtonian fluids; the majority of prior investigations have focused on the prediction of flow characteristics and paid little attention to the accompanying heat transfer aspects [1, 4 to 6]. Despite their widespread use in the food processing industry, the autoclave process for the creation of polymer composites, the dehydration of food suspensions, the filtration of polymer solutions, and the heating/cooling of process streams using tubular heat exchangers, the thermal characteristics of power-law fluids across a periodic array of cylinders remain largely unfunded.

2. Literature review:

Due to economic and environmental considerations, performance enhancement is increasingly necessary in all engineering applications with the goal of maximising the use of available energy and minimising work force. Although there are many industrial uses for tube heat exchangers, their design must consider the limited area. To maximise the total heat transmission (or thermal conductance) between the array and the surrounding fluid, the volume constrained optimisation problem requires determining the appropriate spacing between tubes (or cylinders) of a certain design. One such application of fundamental optimisation accomplishments is the development of cooling techniques for electronic packages. The optimal spacing for forced and free convection in a variety of forms has required extensive investigation [7] following the study published by Bejan *et al.* [8] on the optimisation of arrays of circular cylinders in natural

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convection. Stanescus *et al*. [7] reported the ideal distance between circular cylinders in forced convection with free stream cross-flow. In all investigations, equilateral triangle staggered configurations were taken into consideration.

In addition, to cooling techniques for electronics, the design of heat exchangers with specific novel shapes of geometries, and some theoretical interest in developing correlations for flow and heat transfer for different arrangements, a few notable applications of such non-Newtonian fluid flow past solid objects include flow in polymer processing applications, support structures exposed to the flow stream of fluids. Despite their widespread application in food processing, polymer composites autoclave, food moisture suspensions, filtration of polymer solutions, and process stream heating via tubular heat exchangers, the study of the thermal properties of power-law fluids across a periodic array of cylinders is starving. The discussions above clearly demonstrate that, although there are currently few studies in the literature on non-Newtonian fluids across an array of cylinders and/or over tube banks, there is a wealth of information on the fluid flow and thermal properties of Newtonian fluids across a periodic array of cylinders [1]. Asif and Dhiman [9] investigated the momentum and thermal characteristics of Newtonian fluids for the range of Reynolds number ($10 \leq Re \leq 100$), Prandtl number ($0.7 \leq Pr \leq 50$), and fluid volume fractions (0.70-0.99) using the triangular array of cylinders. The numerical outcomes demonstrated that the parameters taken into account significantly affected the streamlines, isotherms, drag coefficients, and Nusselt numbers. This study is interesting because it is the first to examine the thermal properties of non-Newtonian fluids using an array of circular cylinders. In conclusion, there is relatively little information available regarding the properties of power-law fluids capacity to transport heat across arrays of cylinders. Examining the forced convection thermal characteristics of power-law fluids under steady, laminar, and two-dimensional (2-D) flow conditions through a triangular array of circular cylinders is the goal of this work. A numerical analysis has been made to predict the nature of streamlines and isotherm profiles, variation of thermal heat conductance with power-law index at different Reynolds number (Re) and Prandtl number (Pr) for S/D ranging from 0.5 to 2 and the other governing parameters considered for the current work are in the range of Reynolds number, $40 \leq Re \leq 100$; Prandtl number, $1 \leq Pr \leq 100$; and power-law index, $0.2 \leq n \leq 1.3$.

3. Problem formulation:

The problem under consideration is the free stream with power-law fluids in a channel consist an array of cylinders in cross flow. In the laminar regime, the fluid flow through a bank of cylinders, which are at high temperature T_w, heats up the incoming fluid flowing with velocity U_∞, and temperature T_∞ can be simulated accurately by considering the flow through a single channel such as illustrated by the unit cell in Figure 1. Because of geometric symmetries, there is no

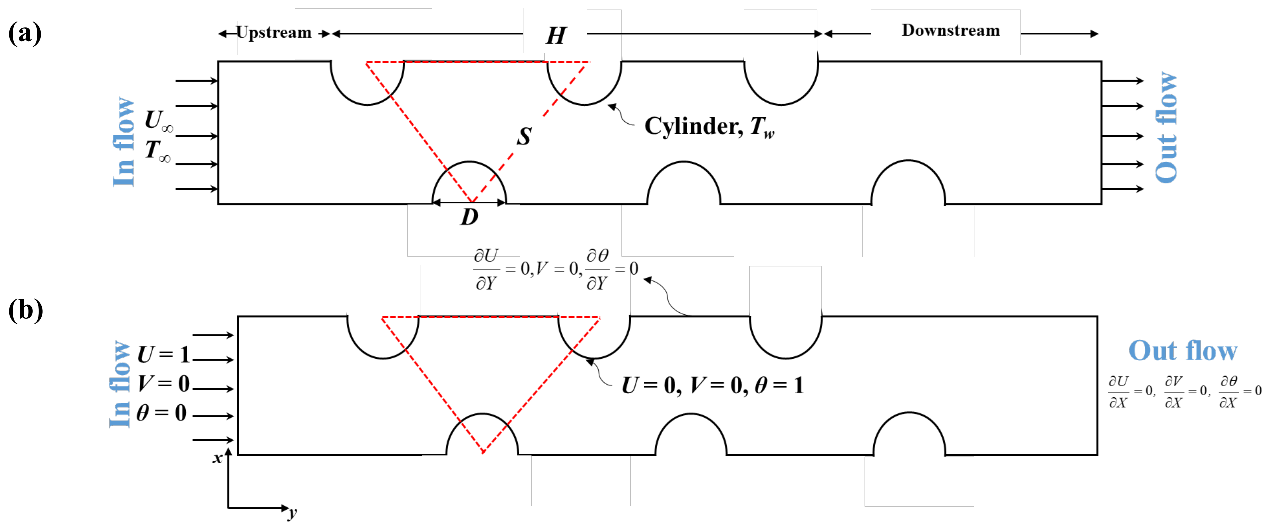

Fig. 1: Schematic representation of (a) physical (b) computational of the problem.

fluid exchange and no heat transfer between adjacent channels. Figure 1, shows H is the channel length and S is cylinder-to-cylinder spacing, and D is the diameter of the cylinders. Here, the computational domain contains the actual channel (flow length H) with an upstream section and downstream section. The problem under consideration is about the geometry with different spacing between the cylinder-to-cylinder in the tube, and this cylinder arrangement is in the equilateral triangular array. The diameter (D) of cylinders are equal for all the conditions and S is the spacing between cylinders, which is likely to optimize in order to obtain the maximum heat transfer or thermal heat conduction. Here, the ratio $H/D = 10$ taken into consideration. Firstly, a certain spacing between the cylinders for which the heat transfer rate between the cylinder and the incoming fluid taking for consideration for Newtonian fluid is maximized and then that optimal geometry obtained is analysed thermally for power-law fluids and the heat transfer rate for those is compared with Newtonian fluids. The study has been conducted based on the assumptions: 2-D steady state, laminar flow, incompressible nature of power-law fluids, all thermo-physical properties are constant. The non-dimensional forms of mass, momentum and energy equations are written as:

Continuity equation:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

Momentum equations:

x component:

$$\left(U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}\right)=-\frac{\partial P}{\partial X}+\frac{1}{Re}\left(\frac{\partial \tau_{xx}}{\partial X}+\frac{\partial \tau_{yx}}{\partial Y}\right) \tag{2}$$

y component:

$$\left(U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial y}\right)=-\frac{1}{\rho}\frac{\partial P}{\partial Y}+\frac{1}{Re}\left(\frac{\partial \tau_{xy}}{\partial X}+\frac{\partial \tau_{yy}}{\partial Y}\right) \tag{3}$$

Energy equation:

$$\left(U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}\right)=\frac{1}{RePr}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{4}$$

The non-dimensional constitutive equation of the power-law model:

$$\tau = m\dot{\gamma}^n \tag{5}$$

where, m = fluid consistency index, and n = power-law index. For incompressible power-law fluids, τ_{ij} are the dimensionless shear stress component of the deviatoric stress tensor, τ where $(i, j) = (x, y)$ is written as

$$\tau_{ij} = -\eta\varepsilon_{ij} \tag{6}$$

where, the non-dimensional power-law viscosity, $\eta = I_2^{(n-1)/2}$, I_2 = second invariant of the rate of strain tensor, and ε_{ij} are the dimensionless component of the shear rate tensor. The physical realistic boundary conditions for the present problem are as follows:

At the inlet: $U = 1$, $V = 0$, $\theta = 0$

At the outlet: $\frac{\partial U}{\partial X}=0,\ \frac{\partial V}{\partial X}=0,\ \frac{\partial \theta}{\partial X}=0$

At the surface of cylinder: $U = 0$, $V = 0$, $\theta = 1$

At the symmetry: $\frac{\partial U}{\partial Y}=0,\ V=0,\ \frac{\partial \theta}{\partial Y}=0$

To account for a considerable range in the parameters and expand the scope of the study, the system parameters are non-dimensionalised using various scaling factors. The following is the non-dimensionalization:

$$(X,Y)=\frac{(x,y)}{D},\ (U,V)=\frac{(u,v)}{U_\infty},\ P=\frac{p}{\rho U_\infty^2},\ \theta=\frac{T-T_\infty}{T_w-T_\infty},\ \tau_{ij}=\frac{\tau}{m\left(\frac{U_\infty}{D}\right)^n}$$

where, (x,y) are the Cartesian coordinates, m; p the pressure, N/m^2; ρ the fluid density, kg/m^3; U_∞ the free stream velocity, m/s; (u,v) the fluid velocities, m/s; T the fluid temperature, K; T_∞ the free stream temperature, K; T_w the cylinder surface temperature, K; D the diameter of the cylinder, m respectively. Dimensional considerations of this flow suggest that the detailed kinematics is governed by three non-dimensional parameters, namely the Reynolds number, $Re=\frac{\rho D^n U_\infty^{2-n}}{m}$; the Prandtl number, $Pr=\frac{C_p m}{k}\left(\frac{U_\infty}{D}\right)^{n-1}$, overall thermal conductance or volumetric heat transfer density, $q=\frac{q'/(T_w-T_\infty)}{kHLW/D^2}$.

4. Numerical methodology:

The governing equations (1)-(4) along with the realistic boundary conditions are solved using commercial software COMSOL Multiphysics (Version 5.3a) based on the finite-element method. The precision and reliability of the chosen numerical scheme purely depend on the numerical parameters chosen for the problem, i.e., domain size, type of grid, grid elements. The domain-independent test was conducted for upstream and downstream lengths of the channel to assure the results will be free from the inlet and outlet flows in the channel. It was found that the further increase in the lengths of upstream and downstream have a marginal effect on the results of overall thermal conductance. Furthermore, the computational domain is discretised into small quadrilateral and free triangular elements to capture the steepest gradient of velocity and temperature near the walls. The grid test has been performed at the extreme values of the parameters, i.e., n = 0.2 and 1.3, Re = 200 and Pr = 100 for S/D = 0.5 and 2. The three grids G1, G2 and G3 have been examined by gradually increasing the number of elements on the cylinder surface. The resulting total number of elements in the domain are 100603, 112524, and 133315 for G1, G2 and G3, respectively in case of S/D = 0.5 and 52738, 61823, and 70778 in case of S/D = 2. Table 1 shows the result of grid independence test for two geometries with extreme values of spacing between the cylinders i.e. S/D = 0.5 and 2. Furthermore, the results of heat conductance have changes less than 1% on further increase in the grid elements from G1 to G2. In addition, domain and grid, the convergence criteria of 10^{-5} was fixed for the velocity and temperature field, which has marginal differences on the results on further decrease.

Table 1: Grid independence test (Re = 200, Pr = 100, H/D = 10).

Grid	S/D = 0.5				S/D = 2			
	N_T	N_C	n = 0.2	n = 1.3	N_T	N_C	n = 0.2	n = 1.3
			q				q	
G1	100603	200	39.386	19.514	52738	200	25.507	11.308
G2	112524	250	38.558	19.484	61823	250	24.750	11.358
G3	133315	300	38.387	19.449	70778	300	24.674	11.374

5. Result and discussion:

5.1 Validation:

It is customary in practice to check the reliability of numerical parameters and model to ascertain the level of accuracy of new results. For that, the validation has been done by comparing the results for available limiting cases of the present study in the literature. Figure 2 illustrates the overall heat conductance, q of Stanescu *et al*. [7] that has been compared with the present results for the case of a cylinder arrangement is equilateral triangular in a Newtonian fluids. Furthermore, Figure 3 compares the overall thermal conductance at Re = 100, 150, and 200, Pr = 0.72 and H/D = 6.2 with that of Matos *et al*. [10]. The results shows good arrangement with each other. Finally, the optimal spacing between cylinder-to-cylinder for circular cylinder for Pr = 0.72 and H/D = 6.2 have been compared with that by Stanescu *et al.*

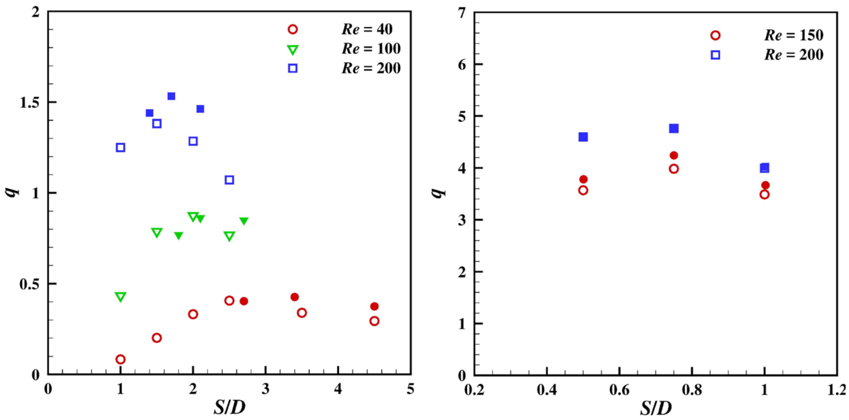

Fig. 2: Comparison of overall thermal conductance with spacing of cylinders present study (hollow symbol) and Stanescu *et al*. [7] (filled symbol).

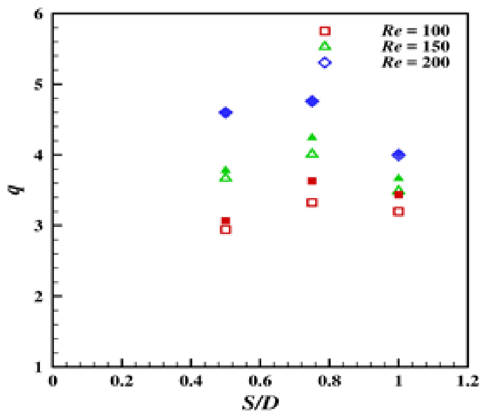

Fig. 3: Comparison of overall thermal conductance with spacing of cylinders present study (hollow symbol) and Matos *et al*. [10] (filled symbol).

[7] in Table 2. Both sets seems in good corresponding to each other. Such close agreements in Figures 2, 3 and Table 1 inspire confidence in the precision and dependability of the new findings in the present work.

Table 2: Comparison of optimal spacing between cylinder-to-cylinder.

Re	S_{opt}/D	
	Stanescu et al. [7]	**Present**
40	2.5	2.5
100	1.9	2.0
200	1.6	1.5

5.2 Thermal conductance:

Figure 4 illustrates variation of overall thermal conductance, q with power-law index, n for the range of the Reynolds number and the Prandtl number for $S/D = 0.5$ and 2. Figure shows that overall heat conductance is higher for higher value of the Reynolds number for shear-thinning fluids for all the cases of Prandtl number i.e. $Pr = 1$, 7, and 100 and decreases

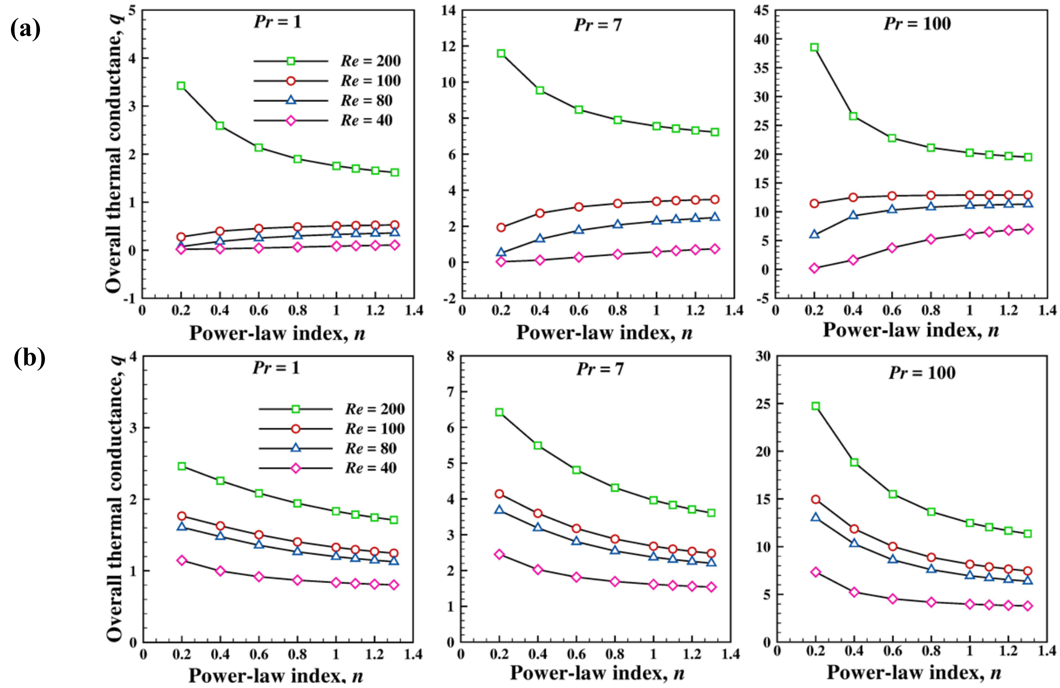

Fig. 4: Effect of power-law index (n) on the overall thermal conductance (q) over the range of Reynolds number and Prandtl number for (a) $S/D = 0.5$ (b) $S/D = 2$.

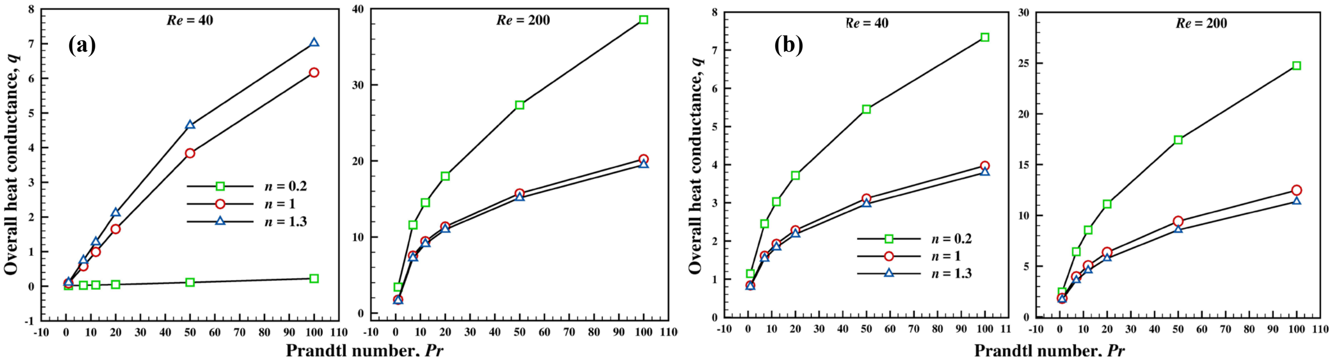

Fig. 5: Effect of Prandtl number (Pr) on the overall heat conductance (q) over the range of power-law index and Reynolds number for (a) $S/D = 0.5$ (b) $S/D = 2$.

as power-law index increases. Figure 5 illustrates variation of overall thermal conductance, q with Prandtl number, Pr for the range of the Reynolds number and the power-law index for $S/D = 0.5$ and 2. Figure shows that the overall thermal conductance, q for shear-thickening fluids is higher on further increasing the Prandtl number for a particular value of the Reynolds number i.e., $Re = 40$ for smaller cylinder-to-cylinder spacing. But for higher cylinder-to-cylinder spacing it is seen that for shear thinning fluids the higher overall thermal conductance is observed as Reynolds number increases from $Re = 40$ to 200.

5.3 Flow kinematics and thermal field:

Figure 6 and 7 shows illustrated the flow (streamlines: with lines) and thermal (isotherms: with color contours) fields near cylinder at the extreme values of Re and Pr for $S/D = 0.5$ and 2, the effect of parameters on the overall heat

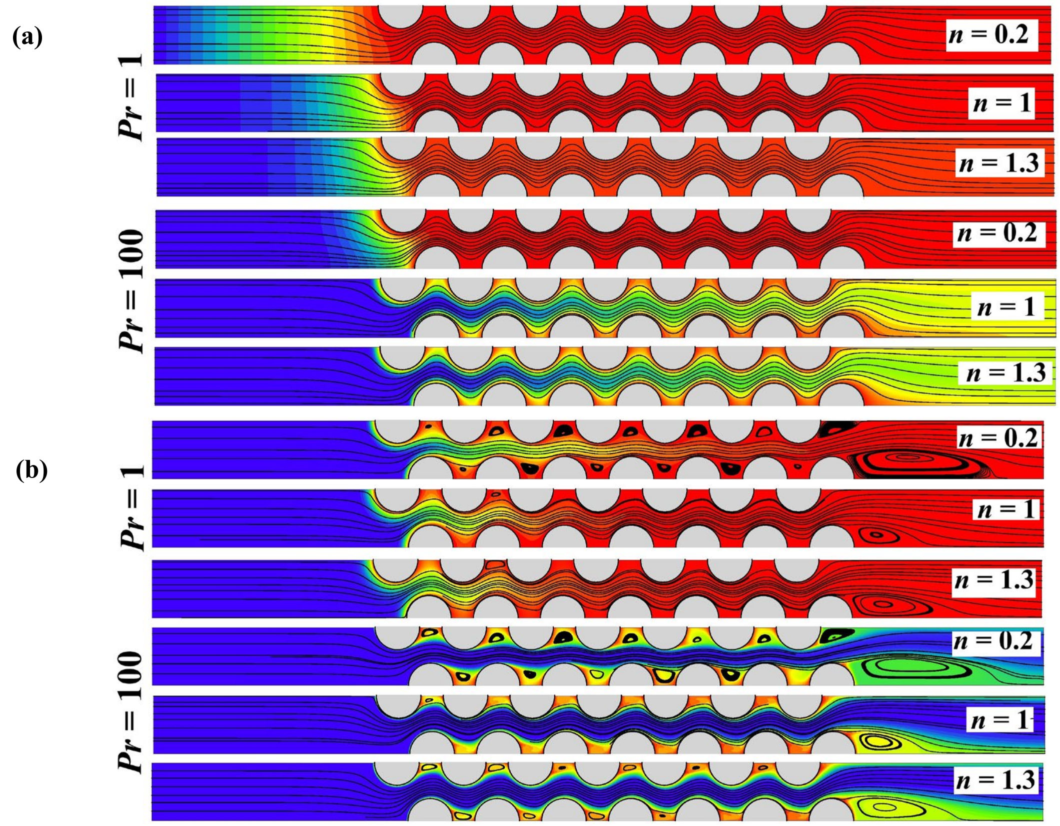

Fig. 6: Streamline patterns and temperature isotherms at (a) $Re = 40$ (b) $Re = 200$ for $S/D = 0.5$.

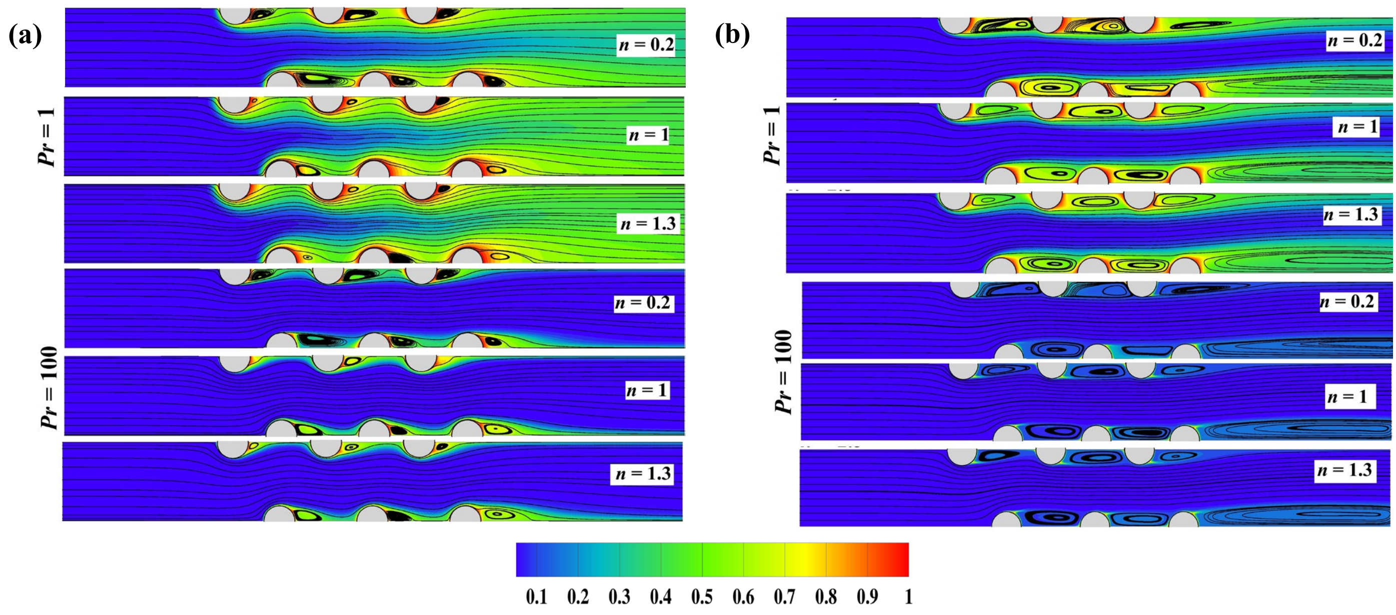

Fig. 7: Streamline patterns and temperature isotherms at (a) $Re = 40$ (b) $Re = 200$ for $S/D = 2$.

conductance, q. The effect of power-law index, n and Prandtl number, Pr on streamline patterns is shown in the figure. For $S/D = 0.2$, there is no vortex formation for $Pr = 1$ from $n = 0.2$ to 1.3 at $Re = 40$, and, vortex formation for $S/D = 2$

is observed for n = 0.2 to 1.3 at Re = 200 for both values of Prandtl number considered here. Also, for Pr = 100 vortex formation is lengthen in the downstream for n = 0.2 at Re = 200. Therefore, we can say that the centre of the vortex formation is changing for different values of power-law index, n observed for higher value of Reynolds number. On further increasing the cylinder-to-cylinder spacing, i.e., S/D = 2, vortex formation is higher for higher value of Reynolds number i.e. Re = 200 for shear thickening fluids, i.e., n = 1.3 and Pr = 100.

In addition, Figures with extreme Prandtl numbers (Pr = 1, 100), respectively, show how isotherms vary with Reynolds number, power-law index and cylinder-to-cylinder spacing. It has been discovered that for higher Reynolds number and Prandtl number, the effect of S/D on isotherm is more significant. For all values of n, S/D, and Pr, the isotherm appeared to be denser towards the cylinder at higher Re. In addition, when Reynolds number grows, increasingly steeper temperature gradients are observed closer to the cylinders. The rate of heat transmission will increase or decrease depending on whether the temperature gradient is steep or weak. Furthermore, when S/D = 0.5, the effect of power-law index (n) on isotherm contours is more apparent at higher Re and lower Pr. Additionally, when we travel towards the downstream region of the domain for lower Pr and Re, heat transfer decreases as the fluid flow character changes from shear-thickening to shear-thinning. This occurs because, compared to similar Newtonian and shear-thickening fluids, the thermal boundary layer in shear-thinning fluids is thicker. When Re = 40 and 200 are low Pr, a sharp thermal gradient results from the transition from shear-thinning fluids to shear-thickening fluids, which leads to an increase in heat transfer rate. This is true for S/D = 2, where the thermal boundary layer is higher close to the cylinder walls.

6. Conclusion:

The range of parameters considered in this study are $40 \leq Re \leq 200$; $0.2 \leq n \leq 1.3$; $1 \leq Pr \leq 100$ and detailed analysis of thermal heat conductance, streamlines and isotherms are shown for the two cylinder-to-cylinder spacing, i.e., S/D = 0.5 and 2. Streamline patterns are shown for the extreme values of parameters and it is influenced by cylinder-to-cylinder spacing at low Prandtl number and power-law index values over the range of Reynolds number. The increasing value of Prandtl number, power-law index and cylinder-to cylinder spacing have tendency to lengthen the streamline pattern and vortex formation. The heat transfer conductance have dramatic change at high Prandtl number and low value of power-law index and increases with the increase in Reynolds number. Similarly, increase in cylinder-to-cylinder spacing, the thermal heat conductance increases for parameters taken into account.

Nomenclature

C_p	Specific heat of fluid	[J/kg.K]
D	Diameter of cylinder	[m]
H	Length of channel	[m]
k	Thermal conductivity	[W/mK]
m	Flow consistency index	[Pa.s^n]
n	Power-law index	[dimensionless]
Pr	Prandtl number	[dimensionless]
p	Pressure at the surface of cylinder	[Pa]
q'	Heat transfer rate	[J/s]
q	Overall thermal conductance	[W/mK]
Re	Reynolds number	[dimensionless]
S	Spacing between the cylinders	[m]
T_w	Temperature of cylinder	[T]
T_∞	Temperature at free-stream	[K]
T	Temperature of fluid	[K]
U_∞	Velocity of free-stream	[m/s]
u,v	Velocity components	[m/s]
W	Height of channel	[m]
x,y	Cartesian coordinates	[dimensionless]
Greek symbols		
μ	Viscosity of fluid	[Pa.s]
v	Kinematic Viscosity	[Pa.s]
γ	Shear rate	[s^{-1}]
ρ	Density of fluid	[kg/m^3]

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