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Thermal Analysis of the Effect of Non-Newtonian Fluid on the Optimally Spaced Elliptical Cylinder

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Abstract - The transfer of heat between bluff bodies and working fluids is a critical factor in various industrial applications, such as the design of heat exchangers and chemical reactors that can significantly impact the efficiency and performance of the system. This study aims to explore the heat transfer density between two aligned elliptical cylinders and a power-law fluid under external forced convection. The study investigates the influence of the Bejan number ($100 \leq Be \leq 10000$) and Prandtl number ($1 \leq Pr \leq 100$) for different aspect ratios ($r = 0.1, 1, 5$) of elliptical cylinders. Extensive results of streamlines and isothermal contours are discussed to delineate the influence of range of parameters on the flow and thermal field around the elliptical cylinders. The study also examines the heat transfer densities as a function of the Be, Pr, n and r. The results reveal that the heat transfer densities have a complex dependency on Be and Pr, particularly for shear-thinning fluid ($n < 1$). However, for shear-thickening and Newtonian fluids ($n \geq 1$), the heat transfer densities increase with the increase in both Prandtl and Bejan number.

***Keywords*:** Elliptical cylinder, power-law fluid, Bejan theory, heat transfer density, Prandtl number

1. Introduction

The optimization of industrial processes for maximum energy utilization is a growing scientific research field. With increasing energy demand, intelligent use of energy is crucial. Heat exchangers, used in air conditioning, refrigeration, heaters, and radiators, must be designed based on space availability. The goal is to find a configuration that maximizes heat transfer density. Economic and environmental concerns necessitate improving performance in engineering applications, particularly tube heat exchangers. These tubes must be sized based on space availability, with various layouts available. Heat exchanger designers require sufficient data to select optimal designs.

Convection heat transfer from circular cylinders in crossflow has been studied extensively, with studies showing that tube arrangement can significantly impact heat transfer intensity. Experimental studies have found that the local heat transfer coefficient distribution depends on tube row location. Several studies have investigated the optimal spacing of circular cylinders in free-stream crossflow forced convection [1], and theoretical analyses [2] have shed light on forced convection heat transfer for incompressible Newtonian fluids. Additionally, studies have shown that staggered cylinders can significantly increase the maximum average Nusselt number [3], while decreasing aspect ratios can lead to higher average Nusselt numbers [4]. Berbish [3] revealed that staggering four cylinders can result in an 85% increase in the maximum average Nusselt number compared to a single cylinder. Sanitjai and Goldstein's [4] have also identified three distinct flow regions around a circular cylinder, including the laminar boundary layer, reattachment of the shear layer, and periodic vortex flow region. Using a finite volume method, Bharti et al. [5] investigated the effect of Reynolds and Prandtl number on the heat transfer in the crossflow of fluid from a heated circular cylinder under forced convection. Their numerical findings suggests that the average Nusselt number increased significantly with increasing the Reynolds and Prandtl number.

The study of heat transfer from non-circular tubes, especially elliptic tubes, is currently underway with the aim of developing more efficient and compact heat exchangers. Experimental studies by Ota et al.[6] and Nishiyama et al.[7,8] have shown that elliptic tubes have a distinct local heat transfer coefficient when compared to circular cylinders, with the angle of attack and cylinder spacing playing a significant role in tandem arrangement. It has been found through various studies that elliptical tubes improve the heat transfer rate and reduces the pressure drop up to 30% than circular tubes/cylinder. Matos et al. [9] conducted a comprehensive analysis on circular and elliptic tube heat exchangers, which revealed that the optimal elliptical arrangement can result in a relative heat transfer gain of up to 13%. Furthermore, they presented a three-dimensional numerical and experimental optimization for staggered arrangements of finned circular and elliptic tubes in forced convection, which showed a significant 19% gain in heat transfer. Razera et al. [10] explored a study on the impact of spacing between elliptical cylinders to enhance the heat transfer rate in forced convection for Newtonian fluid at a fixed Prandtl number. Suffice it to say that adequate information is now available on most aspects of optimization of heat transfer from the staggered or bundle of circular and elliptical tubes in Newtonian fluids.

In industrial practice, substance with multi-phase nature and high molecular weight (pulp and paper suspensions, food, polymer melts, solutions and in biological process engineering applications, etc.) often exhibit shear-thinning and/or shear-thickening behaviour. Bharti et al. [11] conducted a thorough study on the flow past a single elliptical cylinder in power-law fluid in cross flow forced convection, covering the range from shear thinning to shear thickening behaviour. The study revealed that heat transfer is much more facilitated in the case of shear thinning, while it is impeded in shear thickening fluid compared to Newtonian fluid. Klein et al. [12] studied the row of elliptical tubes in shear thinning fluid for fixed Bejan ($Be = 10^5$) and Prandtl number ($Pr = 1$) using constructal design theory focusing on the maximum dimensionless heat transfer density. The study suggested that the optimal aspect ratio increases with power-law index (n), higher heat transfer density is observed for more shear-thinning fluids, and slender tubes perform better in terms of heat transfer. The present work aims at extending the constructal design to non-Newtonian fluids for understanding the effect of Bejan number and Prandtl number over a wide range of power-law index $0.2 \leq n \leq 1.8$ at an optimal spacing suggested by Razera [10] for different aspect ratios of elliptical cylinder.

2. Paper Format

In this work the optimally spaced elliptical cylinder of aspect ratio $r = a/b$ is studied under the laminar, incompressible and two-dimensional convective cross flow is studied. Fig. 1 represent the computational domain and the boundary conditions of the present work. Fluid is driven by the pressure difference between the inlet (P_o) and the outlet ($P = 0$) boundary while cylinder is kept at no slip boundary condition. For the heat transfer study, the surface of the cylinder is kept at constant higher temperature (T_h) than the incoming fluid (T_c). All the thermophysical quantities are assumed to be constant for this work. Three different aspect ratios ($r = 0.1, 1, 5$) of elliptical cylinder are considered in the work that are optimally spaced at distance So from the other cylinder. The spacing between the cylinder are kept similar to Razera et al. [10] for different Bejan number and aspect ratio.

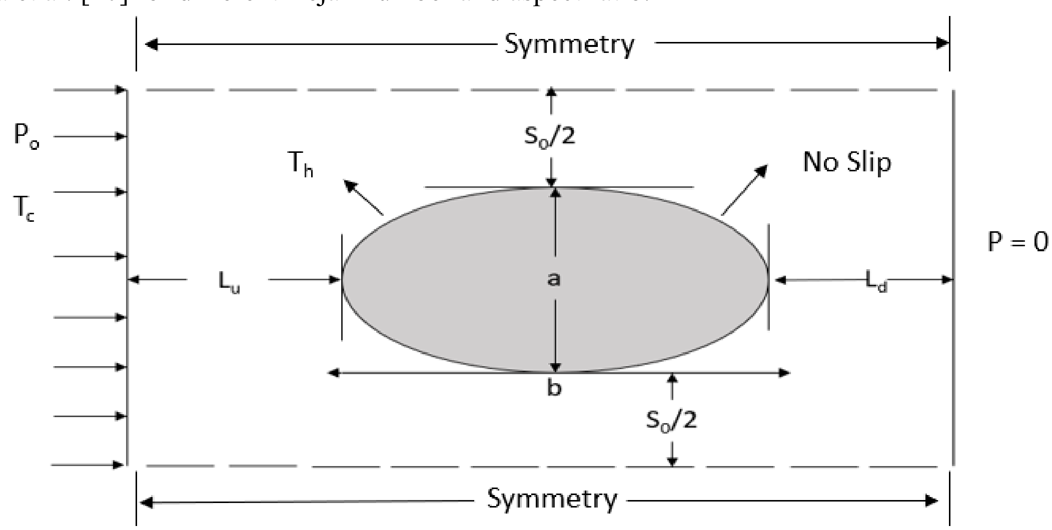

Fig. 1: Schematic representation of the computational domain.

The non-dimensional form for continuity, momentum and energy equation for the steady, laminar and convective flow are written as follow:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \quad (1)$$

$$\frac{Be_A}{Pr}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)+\frac{\partial P}{\partial x}-\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)=0 \quad (2)$$

$$\frac{Be_A}{Pr}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)+\frac{\partial P}{\partial y}-\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)=0 \quad (3)$$

$$Be_A\left(u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}\right)-\left(\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}\right)=0 \quad (4)$$

The spatial coordinate, velocity and pressure are non-dimensionalized by $A^{1/2}$, $\Delta PA^{1/2}/\eta$, ΔP variables respectively. The non-dimensional temperature θ is given by $T-T_c/T_h-T$. Non-Newtonian fluid properties are defined by power-law model in this work and the apparent viscosity of power-law fluid is written as: $\eta = m\dot{\gamma}^{n-1}$, where m and n is the consistency index and power-law index respectively.

The non-dimensional number that are used in this work can be given as follows:

Bejan number: $$Be_A = \Delta PA/\eta\alpha \quad (5)$$

where ΔP is the pressure difference between inlet and outlet, A is the cross-sectional area of the cylinder, η is the apparent viscosity of the power-law fluid and α is the thermal diffusivity of the fluid.

Prandtl number: $$Pr = \eta C_p/k \quad (6)$$

where η is the apparent viscosity of the power-law fluid, C_p is the specific heat and k is the thermal conductivity of the fluid.

The heat transfer density is given by:

$$q = q' \Big/ \frac{b(a+S)}{A}k(T_h-T_c) \quad (7)$$

where S is the spacing between the cylinder and q' is the rate of heat transfer per unit length of the cylinder.

Table 1: Grid independence test

r = 1						
	Be = 100, S = 1.9			**Be = 10000, S = 0.55**		
NC	**ND**	**q**, **n = 0.2**	**q**, **n = 1.8**	**ND**	**q**, **n = 0.2**	**q**, **n = 1.8**
120	50848	0.351	1.725	110859	3.887	10.722
240	61514	0.351	1.725	115059	3.965	10.715
480	76908	0.351	1.725	114171	3.992	10.715
r = 5						
	Be = 100, S = 1.3			**Be = 10000, S = 0.6**		
NC	**ND**	**q**, **n = 0.2**	**q**, **n = 1.8**	**ND**	**q**, **n = 0.2**	**q**, **n = 1.8**
120	19181	3.137	4.438	22845	19.894	13.607
240	21627	3.138	4.438	25193	19.795	13.595
480	28033	3.145	4.438	31407	19.789	13.572

3. Numerical Methodology

The governing equations (1)-(4) are solved using finite element based COMSOL Multiphysics 5.3 software. parameters like domain and grid in the domain have strong impact on the evaluated results. Thus, to make sure the results are free from such effect both domain and grid test are performed for the extreme case of geometry as well as parameters. Domain tests are performed by varying the upstream and the downstream length of the domain for both shear-thinning and shear-thickening fluid at two Bejan number, $Be = 100$ and 10000. The value of L_u (1, 6, 20) and L_d (20, 30, 50) are varied. It shows that $L_u = 6$ and $L_d = 30$ was found to be sufficient to consider the results free from any domain effects. Similarly, three different grids are chosen to study the grid effect on the heat transfer density at the extreme value of aspect ratio, r and power-law index, n at highest Pr number as shown in Table 1. Results shows that grid G2 having 240 elements at the surface of cylinder is appeared to be sufficient to capture the thinnest boundary layer. The relative convergence criteria of 10^{-6} is considered in this work.

4. Results and Discussion

The current work evaluates the effect of power-law index, $0.2 \leq n \leq 1.8$ over three aspect ratio of the elliptical cylinder, $r = 0.1$, 1 and 5. The variation of heat transfer is studied for various Bejan number, $Be = 100$, 500, 1000, 5000, 10000 over a wide range of Prandtl number, $0.72 \leq Pr \leq 100$. It is customary to check the present numerical methodology with the previously available literature in order to check the reliability and accuracy of the current results. Figure 2 shows the comparison of heat transfer density over a range of Bejan number and aspect ratio for the present work with the Razera et al. [10] for the Newtonian fluid. The solid line represents the literature data [10] and symbol represent the current results. The results show good correspondence with each other.

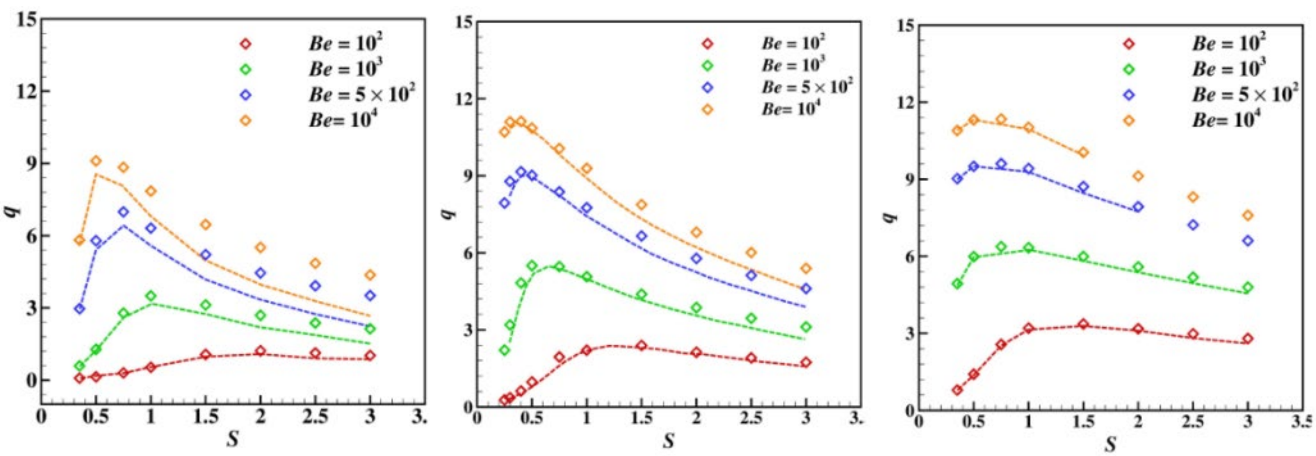

Fig. 2: Comparison of heat transfer density with Razera et al. [10].

4.1. Streamline and Isothermal Contours

Figure 3 and 4 shows the streamline (represent as lines) and isothermal contours (colour contour) for two aspect ratio ($r = 0.1$ and $r = 5$) and extreme values of Be, Pr, and n. The presence of wake formation at the rear end of the cylinder shows the complex dependency of the flow onto the various fluid parameters Be, Pr and n. Broadly, with the increase in the aspect ratio from 0.1 to 5, the recirculation of the flow increases due to blunter shape of the cylinder. Thus, at $Pr = 1$, the wakes are formed over the entire range of power-law index for $r = 5$. But the size of the wake has the inverse dependency on n; it starts to reduce with the increase in the viscous nature of the fluid at the fixed value of Bejan and Prandtl number. The increase in Bejan number enhances the wake formation and make it even more prominent and drastically increase the size of the wake formed which assists the flow circulation and mixing of the fluid. Due to coupling between momentum and energy equation according to Bejan number, the flow fields are also affected with the variation in Prandtl number. In case of fixed Be and n, the size of wake decreases with the increase in Pr, irrespective of the aspect ratio of the cylinder.

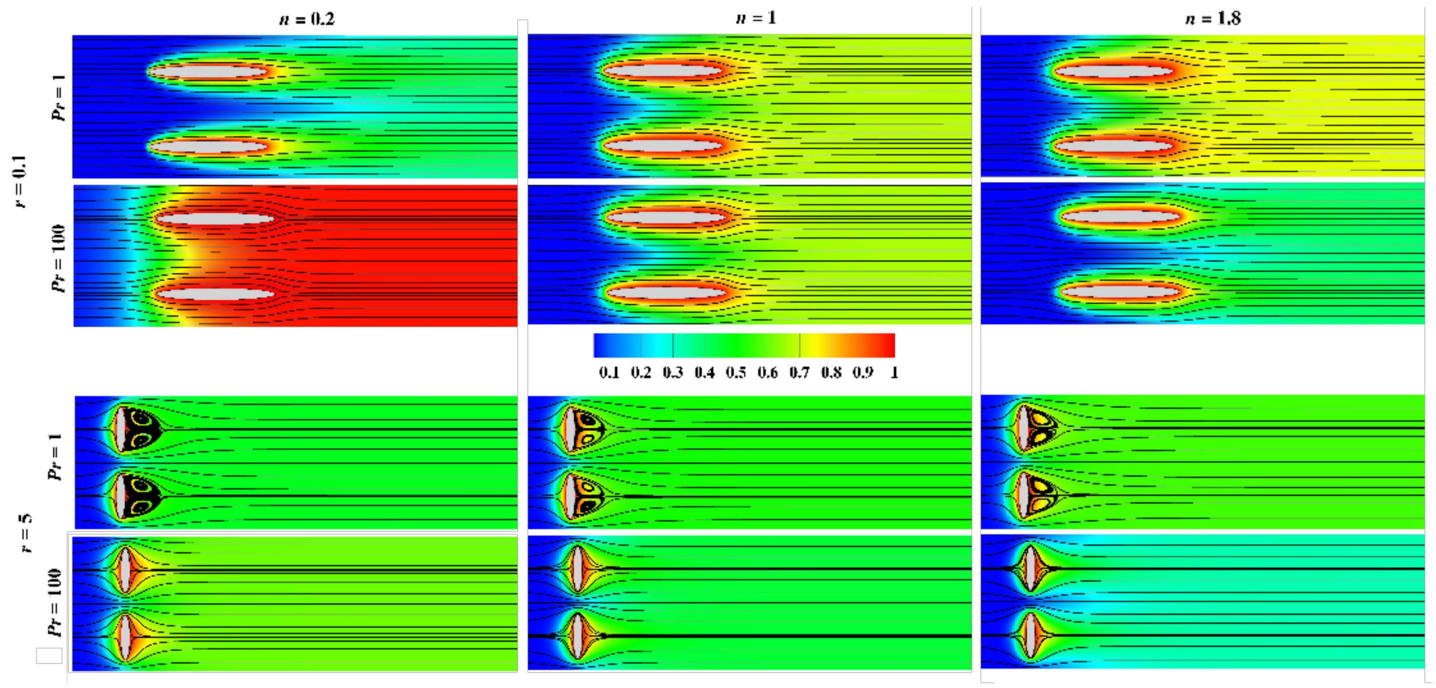

Fig. 3: Streamline and isothermal contours at extreme value of *Pr* and *r* at *Be* = 100

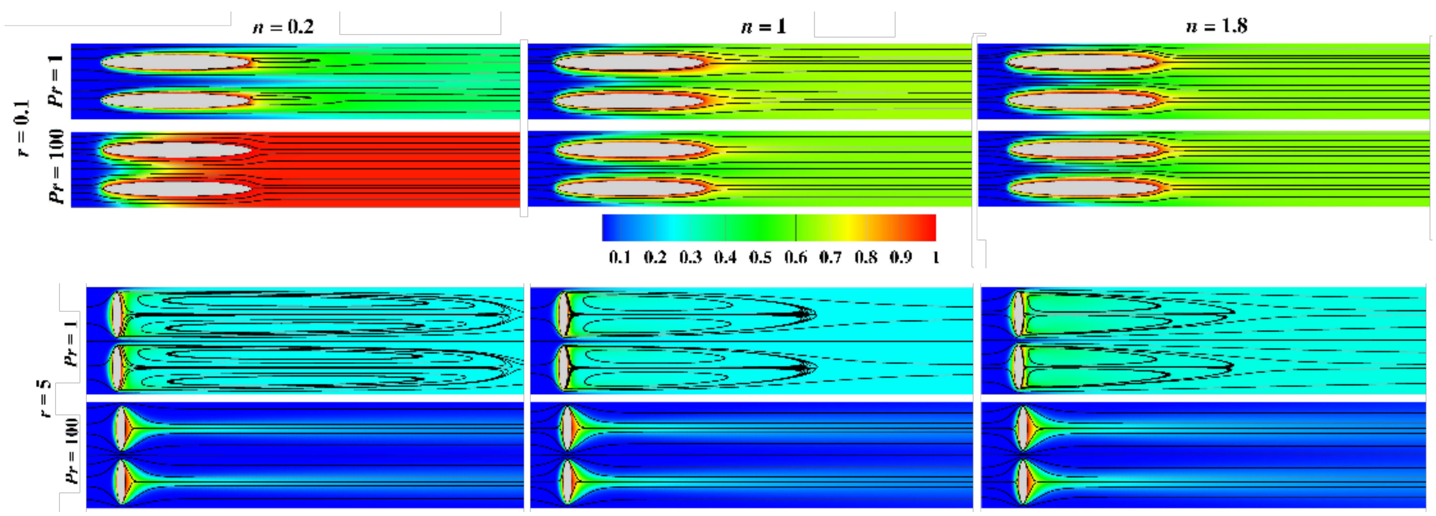

Fig. 4: Streamline and isothermal contours at extreme value of *Pr* and *r* at *Be* = 10000.

Similarly, isotherms are also plotted as the non-dimensional temperature in the domain over the range of parameter. For the fixed value of Bejan number and aspect ratio, for shear thinning fluid the thermal boundary layers get intermix with each other with the increase in Prandtl number which results in low heat transfer rate. This is due to the effect of increasing heat advection with the decrease in the viscosity of the fluid at low power-law index values. However, due to high viscous nature of shear thickening fluid the thermal boundary layer gets thinner with the increase in both n and Pr, the temperature gradient in the flow field improves as shown in Fig. 3 and 4. The increase in Bejan number further decreases the thermal boundary thus improves the rate of heat transfer.

4.2. Heat transfer density

Heat transfer density is the non-dimensional term to represent the heat transfer from the cylinder surface to the surrounding fluid. In Fig. 5 heat transfer density is plotted against the power-law index over a range of Bejan number and

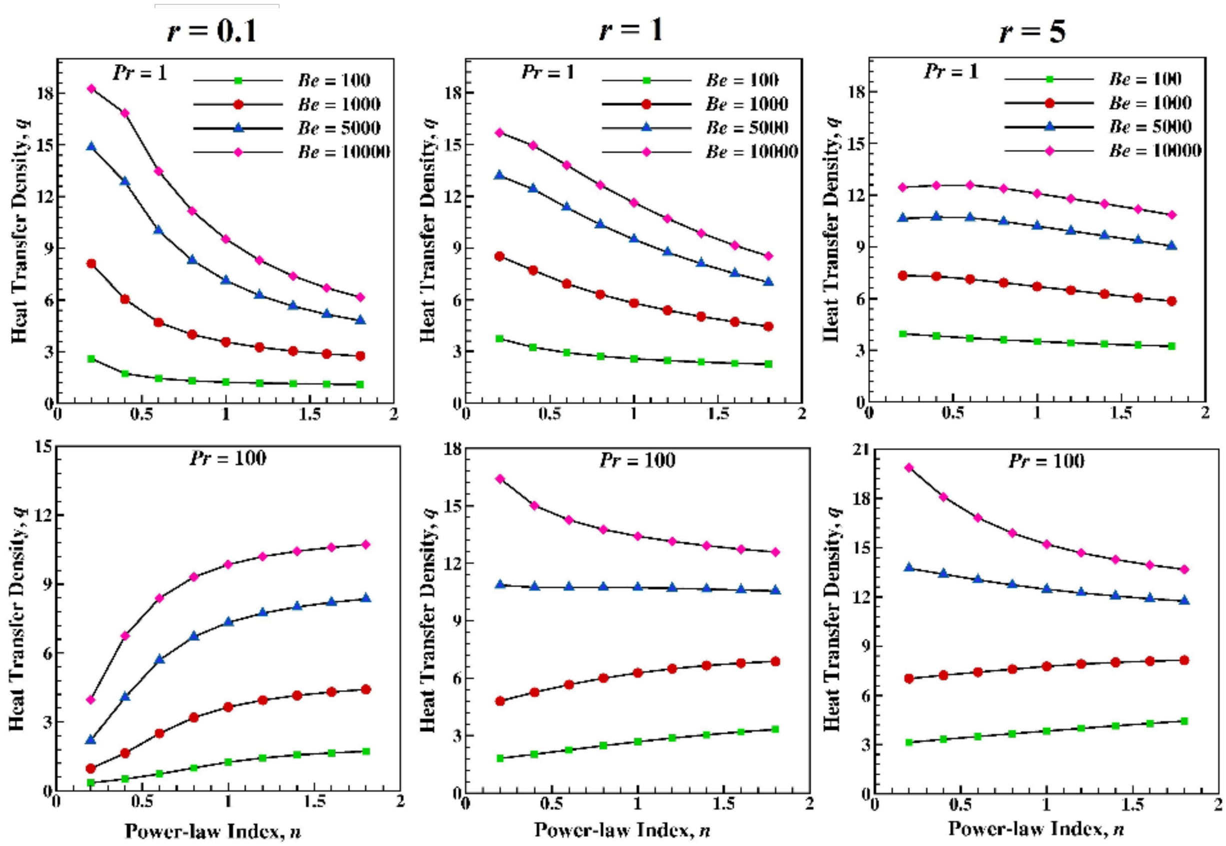

Fig. 5: Effect of *Be* on the heat transfer density for different power-law fluids at the extreme values of *Pr* number for different aspect ratios of cylinder.

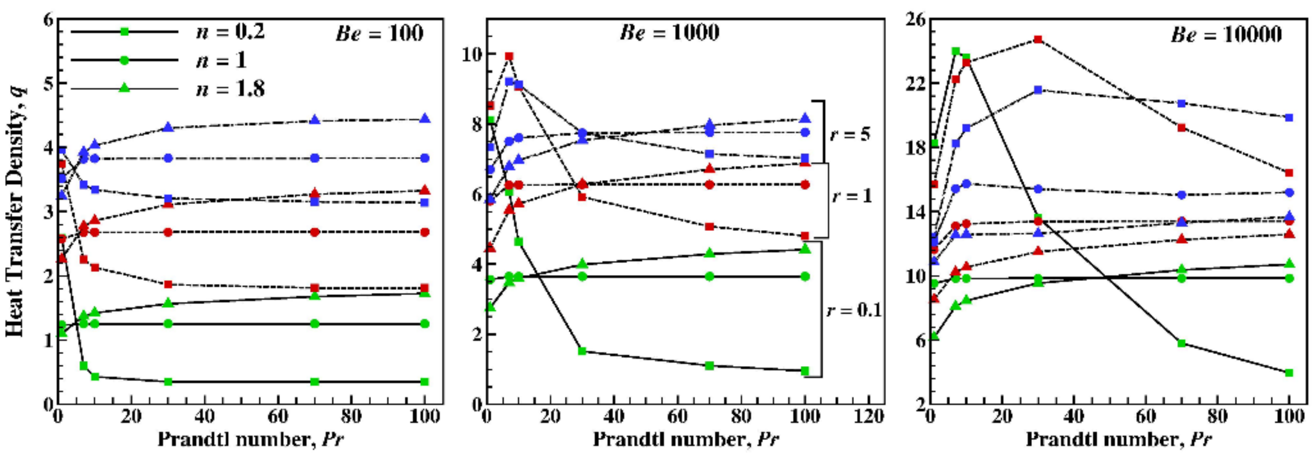

Fig. 6: Comparison of heat transfer density at different aspect ratio and power-law index (a) $Be = 10^2$ (b) $Be = 10^3$ (c) $Be = 10^4$

aspect ratio. Generally, with the increase in the Bejan number, heat transfer densities increase at all values of Prandtl number and aspect ratio. At low Prandtl number heat transfer densities reduce with the increase in power-law index irrespective of the Bejan number and the aspect ratio of the cylinder. However, for r = 0.1, heat transfer reduces significantly at higher Prandtl number due to very high intermixing of thermal boundaries in the fluid especially in case of shear thinning fluids.

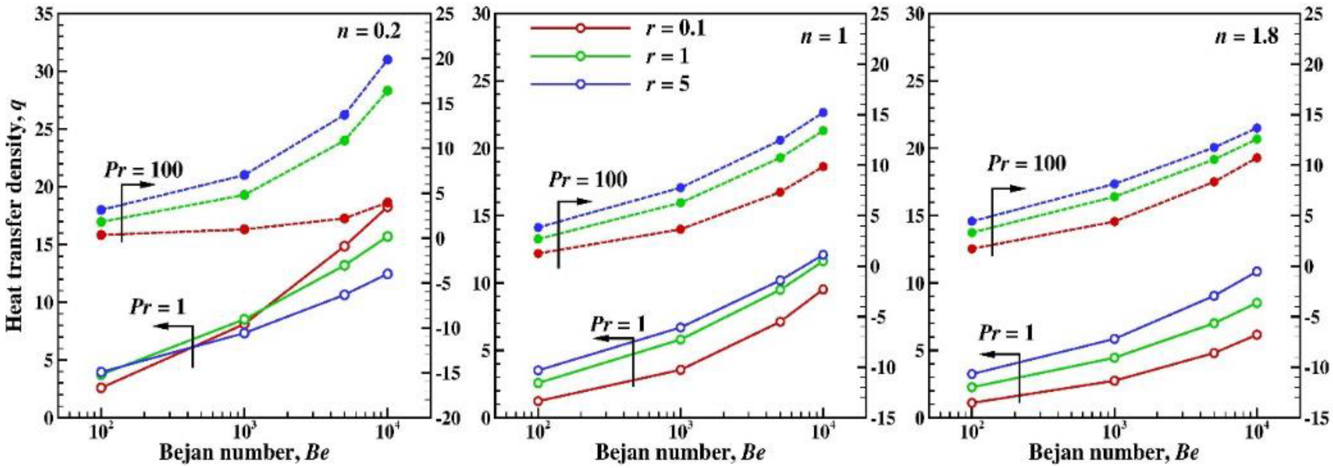

Fig. 7: Variation of heat transfer density with respect to the Bejan number at extreme values of *Pr*, *n* and *r*.

Figure 6 demonstrate that for $n < 1$, heat transfer densities reduce, and $n \geq 1$ heat transfer densities increase with the increase in Prandtl number. It also shows that at low Bejan number r = 5, offers high heat transfer rate over the range of Prandtl number and power-law index. However, r = 0.1 offers high heat transfer densities in case of high Bejan number and low Prandtl number than other aspect ratios. But it starts to reduce with the increase in Prandtl number at a fixed Bejan number. In the literature for the optimum spacing between the elliptical cylinder by Razera et al. [10], it is mentioned that at high aspect ratio, r = 5 elliptical cylinders perform better in case of heat transfer than r = 0.1. Similarly, the heat transfer density is also varied with respect to the Bejan number for fixed value of power-law index in Fig 7. Results shows that for $n \geq 1$, the cylinder of aspect ratio, r = 5 perform better and provide better heat transfer efficiencies. Only in case of low Bejan number and lower power-index value, the reverse trend can be clearly seen in Fig 7.

5. Conclusion

The present work focused on the rate of heat transfer between the elliptical cylinder and power-law fluid over the wide range of parameters: Bejan number ($100 \leq Be \leq 10000$), Prandtl number ($1 \leq Pr \leq 100$) and aspect ratios (r = 0.1, 1, 5) of the elliptical cylinders under the external forced convection. The elliptical cylinders are kept at the optimal spacing for each Bejan number as per suggested in the literature. A detailed velocity and temperature fields are studied in terms of streamlines and isothermal contours. For low Prandtl number particularly for shear thinning fluids r = 0.1 have higher heat transfer densities. But it decreased significantly with the increase in both Prandtl number and power-law index. Consequently, for the optimum spacing suggested in literature only work well for the aspect ratio of r = 5 and showed the better thermal performance in general over the range of Bejan and Prandtl number. The potential of this research is significant for industries that handle non-Newtonian fluids, as it could lead to the development of more efficient heat transfer systems. The study's findings indicate that by enabling a flow system to adapt and modify its degrees of freedom, we can greatly improve its heat removal and fluid flow capabilities.

Nomenclature

A	Cross sectional area of cylinder	m^2
a	Length of vertical axis	m
b	Length of Horizontal axis	m
Be	Bejan number	-
C_p	Specific heat capacity	J/kgK
m	Flow consistency index	$Pa.s^n$
n	Flow behaviour index	-
L	Length of the cylinder	m
p	Pressure	Pa
Pr	Prandtl number	-
q'	Rate of heat transfer	J/s
r	Aspect ratio (=a/b)	-
S	Spacing between the cylinders	m
T_h	Temperature at the surface of the cylinder	K
T_c	Temperature of the fluid	K
Greek Symbol		
η	Apparent viscosity	Pa.s

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