Methodology for Modeling Spray Cooling Of a Cylindrical Tube Heated In the Film Boiling Regime

Nilojendu Banerjee\(^1\), Cameron Tropea\(^2\), Satyanarayanan Seshadri\(^1\)

\(^1\)Department of Applied Mechanics Indian Institute of Technology Madras
Chennai 600036, India
banerjeenilojendu@gmail.com; tropea@sla.tu-darmstadt.de

\(^2\) Technical University Darmstadt Institute of Fluid Mechanics and Aerodynamics
64287 Darmstadt, Germany
satya@iitm.ac.in

Abstract - This work focuses on the physics related to spray cooling of horizontal, circular tubes under film boiling conditions, i.e. when the substrate temperature exceeds the Leidenfrost point. The main objective is to express the steady state heat removal rate from the heated surface, with a focus on understanding heat transfer and drop impact on a complex geometry. The characteristics of the spray, which include drop size, velocity (assumed constant), and spatial distribution, are expressed in terms of local number flux. To account for drop-drop interactions on the cylinder, an effective coverage in terms of coverage efficiency is considered at higher mass flow rates.

Results are presented as the average heat transfer coefficient. The findings highlight the importance of liquid mass flow rate and droplet diameter as the most influential parameters affecting heat transfer. Additionally, the non-normal impact angles caused by the spray angle and the curvature of the cylindrical surface lead to dramatically reduced heat transfer rates. This reduction of the heat transfer can be mitigated by placing multiple nozzles at various longitudinal and circumferential positions around the tube.

Keywords: film boiling, Leidenfrost point, spray cooling, heat transfer

1. Introduction

Heat removal from hot substrates finds applications in numerous fields, including cooling of electronic chips [1], die cooling in forging, quenching of metal [2], or cooling of high-temperature turbine blades. In many processes, the heat transfer rate can be achieved by jet impingement or forced convection cooling [3]. However, for high heat transfer rates an alternative approach such as spray cooling is required, where heat transfer coefficients of up to 1000 - 5000 W/cm\(^2\)K can be achieved [4]. Achieving such high values of heat transfer coefficient is strongly dependent on the temperature of the heated surface. In some cases, this temperature exceeds the Leidenfrost point and the heat transfer coefficient drops rapidly. The reason is that an insulating vapour layer is formed between the surface and the impacting surface [5]. It is clear that the Leidenfrost condition is unfavourable for spray cooling, but in some cases it becomes unavoidable. This occurs in transient cooling when the surface starts at a high temperature. Another situation is when the temperature of the exchanger tube varies along its length, from high to low temperature. This condition above Leidenfrost is also known as film boiling.

The novel aspect employed in the present analysis is that instead of drops impacting onto a flat surface, they impact onto a horizontal cylindrical tube. This change affects the local mass flux of drops as well as the spreading area of the droplet on the surface. Although a similar type of problem has been investigated previously [2], the approach of superimposition of individual drops was not introduced, using instead an empirical approach invoked for the mass flux to estimate the heat transfer.

2. Modelling and Methodology

Figure 1 depicts the simplified geometry involving a horizontal tube with a single spray nozzle placed vertically with respect to the cylinder of radius \(R\) at a distance of \(H\). The maximum opening angle \(y_{max}\) of the spray is selected in such a way that the outermost droplets just make tangential contact with the cylinder. Also, it is assumed that the cylinder exhibits
high thermal inertia, which allows us to consider the cylinder surface to be at constant temperature $T_w$. We can consider the cylindrical surface to be made of an infinite number of small, inclined surfaces $dA_c$ (see Fig 1b). Then the primary objective of the study reduces to the heat transfer of a single drop impact onto an inclined surface at an oblique angle $\psi$. The individual areas $dA_c$ are chosen small and assumed to be flat, although inclined.

**Figure 1:** Pictorial diagram of the spray, tube and the associated coordinate systems.

### 2.1. Heat Transfer from a Single Droplet Impact

The analysis of heat transfer related to a single drop on to a flat surface beyond the Leidenfrost point has been extensively studied by [6]. We have used these results with a modification by incorporating the oblique angle dependence ($\psi$). Accordingly, the heat flux ($q''_{nf}$) can be computed as

$$q''_{nf} = \frac{2g_2e_w(T_w-T_{sat})}{\sqrt{\pi}(g_1 + 2g_2)}$$

where,

$$g_1 = \sqrt{(g_2 - g_3)^2 + 4g_2/\sqrt{\pi} - g_2 - g_3}; \quad g_2 = \sqrt{\frac{5}{\pi}(T_{sat} - T_0)e_l}; \quad g_3 = \sqrt{\frac{k_v\rho_lL}{(T_0 - T_{sat})e_w^2}}$$

In Eq. (1b) $e_w = \sqrt{\rho_w cp_w k_w}$ is the thermal effusivity of the surface and $e_l = \sqrt{\rho_f cp_f k_f}$ is the thermal effusivity of the liquid or fluid. $\rho, cp, k, L$ are defined as density, specific heat capacity, thermal conductivity and heat of vaporization. Similarly $T_w, T_{sat}, T_0$ are the surface, saturation and surrounding temperatures. Here, (w), (f) and (v) are the subscripts denoting surface, fluid and vapour. As heat flux is a vector quantity, it depends on the angle of impact onto the cylindrical tube. The resultant heat flux on the cylindrical tube is thus ($q''_t = q''_{nf} \cos \psi$).

The total heat transfer ($Q_{sd}$) is then expressed as

$$Q_{sd} = \chi \int_0^{t_{in}} q''_c A(t, \psi) dt$$

Here, $t_{in}$ is the time over which heat transfer is significant and is taken as $D/U cos \psi$ [7]. $A(t, \psi)$ is considered as the spreading area of the droplet at the initial time $t_{in}$, given as a function of both time and oblique angle[7], i.e.

Then expression (2) becomes

$$Q_{sd} = \chi \int_0^{t_{in}} q''_c \pi \frac{D'(t, \psi)^2}{4} dt$$

### 2.2. Drop Impact Statistics of Dense Spray Impact

In this section we evaluate the number of droplets that interact with the heated surface per unit time per unit surface area of the cylinder, known as the number flux ($\dot{n}_c$). This depends on the number flux ($\dot{n}_v$) that occurs over an

$$\dot{n}_c = \dot{n}_v \frac{dA_s}{dA_c}$$

where $dA_s = H^2 \sin \gamma dy d\varphi$
elemental portion of the spray \( dA_s \), being a sector of a large sphere of radius \( H \). If we calculate \( \dot{n}_c \) with respect to an elemental surface on the cylinder \( dA_c \) we get,

\[
\dot{n}_c = \frac{2\pi R^2}{\rho \gamma} \frac{N}{\int_0^\infty \frac{\gamma}{\sin \vartheta} d\gamma d\varphi}
\]

where \( \gamma_{\text{max}} = \arcsin \left( \frac{R}{R + H} \right) \) (5b)

In Eq. (5b), \( \dot{n}_c \) can be calculated from

Here, \( N = 6M/\rho \pi D^3 \), where, \( M \) [kg/s] is the spray mass flow rate coming from the nozzle. To find \( \dot{n}_c \) in Eq. (5b), we need to know the value of \( dA_c \). This can be found by considering four diverging lines in space through the corners of \( dA_s \) and intersecting the \( [x = R + H - r \cos \varphi, x = -r \sin \gamma \cos \varphi, y = r \sin \gamma \sin (\varphi)] \)

Inserting the required values from Eqns. (6) and (7) into the Eq. (5), \( \dot{n}_c \) can be evaluated.

In Fig. 2 calculations were performed for two values of \( D = D_0 \) as 100 and 500 \( \mu \)m. Here, \( \varphi = 0^\circ \) and \( \varphi = 90^\circ \) representing the circumferential and the top portions of the cylinder w.r.t spray central axis. The value of \( \dot{n}_c \) decreases dramatically on the line \( \varphi = 0^\circ \) compared to \( \varphi = 90^\circ \) position due to drop oblique impact and surface curvature effects. Also it is observed that \( \dot{n}_c \) decreases with an increase in \( D \), because of less generation of \( N \), which in turn is related to Eq. (6).

Drop impact dynamics on the heated surface is entirely random in nature both in space and time of their occurrence. This phenomenon can be accurately described with the help of a Poisson distribution (Feller et al. [9]). The distribution is represented as

\[
P(n; \lambda) = \exp(-\lambda) \frac{\lambda^n}{n!} \tag{8}
\]

where \( \lambda \) is the relative cumulative area per unit of the observation area, i.e.

\[
\lambda(\gamma, \varphi) = \dot{n}_c(\gamma, \varphi) \int_0^\infty \pi D(t, \psi)^2 / 4 dDdt \tag{9}
\]

Here, we consider all drops in the spray having the same diameter \( D_0 \), then \( p(D) \) becomes a delta function. In Eq. (9) \( D(t, \psi) \) can be expressed with the help of [6] as

\[
D(t, \psi) = 4D_{\text{max}}(t/c - t^2/c^2) \quad \text{With contact time } t_c = \frac{D}{U \cos \psi} \tag{10}
\]

With the help of Eqns. (8) and (9) we can express the effective coverage area ratio, also known as coverage efficiency as

\[
\eta_{\text{cov}} = \frac{1 - e^{-\lambda}}{\lambda} \tag{11}
\]

and the total heat transfer for a single spray on the heated cylindrical tube becomes

\[
\dot{h}_{\text{avg}} = \frac{\int_0^{\pi/2} \int_0^{\gamma_{\text{max}}} \dot{n}_c(\gamma, \varphi) Q_{s\varphi}(\psi) \eta_{\text{cov}}(\gamma, \varphi) dy d\varphi (T_w - T_0)} \tag{12}
\]

3. Results and Discussions

3.1 Validation of the model

To validate the accuracy of the model, the analytical prediction is compared with experimental results from Breitenbach et al. [6] and Wendelstrof et al. [8] for spraying onto a heated flat plate. For this we let the cylinder become a quasi-flat surface by taking \( R = 100 \) m. To evaluate mass flux \( (\dot{m}) \) from the mass flow rate \( (M) \) we consider a particular spray with a maximum spray angle \( (\gamma_{\text{max}}) \) that impacts onto a flat surface in an axisymmetric manner with a ‘wetted’ radius \( R_{FP} = H \tan \gamma_{\text{max}} \). Then mass flux \([ \text{kg/m}^2\text{s} ] \) can then be computed as

\[
\dot{m} = M/\pi R_{FP}^2 \tag{13}
\]

Figure 2. Variation of number flux \( \dot{n}_c \) with dimensionless spray angle \( (\frac{\gamma}{\gamma_{\text{max}}} \cdot R_{FP}/\gamma_{\text{max}}) \): \( R = 0.25 \) m, \( H = 0.3 \) m, \( M = 0.5 \) kg/s.
The agreement shown in Fig. 3 between experiments and prediction can be considered very good, validating the computational approach used in the present code. The small variations can be attributed to the fact that the exact boundary conditions are not known for the experiments.

3.2 Total heat transfer from the tube as a function of droplet diameter and the surface temperature

In Fig. 4a we see that the heat transfer coefficient is significantly higher in the case of small drop diameters. This is due to the fact that with higher drop diameter, the coverage efficiency decreases drastically (see Fig. 3a) which in turn decreases the heat transfer rate. On the other hand, Fig. 4b depicts the same variation with substrate temperature, although this variation is less significant compared to Fig. 4a. This is because at high temperatures, the vapour formation near the surface decreases the heat transfer, but at the same time the surface to surrounding temperature difference increases heat transfer. These two counteracting phenomena reduce the dependence on $T_w$.

![Figure 3](image-url)

**Figure 3.** Computation performed for spray cooling of a flat plate. a) Variation of coverage efficiency ($\eta_{cov}$) with mass flow rate ($M$) for various droplet diameters ($D_0$) and with the parameters $U_0 = 20$ m/s, $H = 0.3$ m, $\gamma_{max} = 27^\circ$.; b) Comparison of heat transfer coefficient with literature values using the parameters $D_0 = 350\mu\text{m}$, $U_0 = 14$ m/s, $\Delta T = 700^\circ\text{C}$ and $\gamma_{max} = 27^\circ$.

![Figure 4](image-url)

**Figure 4.** Computation performed for spray cooling of a cylindrical surface. a) Variation of average heat transfer coefficient ($h_{avg}$) with drop diameter ($D_0$) considering the surface temperature $T_w$ as 500 °C; b) and surface temperature ($T_w$) values using the parameters $D_0 = 100\mu\text{m}$, $U_0 = 20$ m/s. Input parameters are considered as: $\gamma_{max} = 27^\circ$, $R = 0.25$ m and $M = 0.05$ kg/s.

In this study, an analytical prediction has been made for spray impact onto a heated cylinder in the film boiling regime. Both oblique impact and curvature of the cylinder drastically reduce the number flux in the circumferential direction around the cylinder. Drop-drop interactions are considered on the surface of the cylinder as a function of coverage efficiency.

The average heat transfer coefficient is computed as a function of drop diameter and substrate temperature, which shows clearly that a smaller drop diameter and surface temperature yield higher heat transfer from the cylinder.

**References**


