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A Study of Kruppa's Equation for Camera Self-calibration

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Abstract- In this paper we discuss the Kruppa's equation, which is widely known as the first camera self-calibration method. The classical and the simplified version of the method are of our interest. We have analyzed it based on how it is derived. As Kruppa's equation works based on the image of the absolute conic, which is an imaginary conic, we have found that the use of circular points at infinity was missed in the classical Kruppa's in generating the imaginary conic. We have pointed out that the concept of degenerate point conic in deriving the simplified version is one of the reasons of degeneracy. Other than that, we also found some other inaccuracies of the derivation. For the future use as well as the improvement of the method, we have proposed possible solutions for it.

Keywords: Self-calibration, Kruppa's equation, Absolute conic, Circular points at infinity.

1. Introduction

In computer vision, camera calibration plays an important role as a prerequisite, especially for applications where the relationship of 2D images and 3D world is needed. It is a process of finding the intrinsic and extrinsic parameters of the camera. Recovering the intrinsic parts is called Self-calibration (or Auto-calibration), which only depends on the given "uncalibrated" images. The term Self-calibration itself was first introduced by Maybank and Faugeras (1992). They have presented a work on Self-calibration based on the relationship between the intrinsic camera parameters and the epipolar transformation, associated with the camera displacements. The theoretical background was previously discussed in detail by Faugeras et al. (1992), where in the paper it is mentioned that the whole camera calibration process can be initiated by finding the intrinsic camera parameters and then the extrinsic parameters can be estimated afterwards. In the same paper, they have introduced the use of the Kruppa's equation for camera self-calibration.

Later on, the Kruppa's equation were derived in different ways from it was first introduced. For example the derivation based on the cross ratio can be found in the work of Maybank et al. (1992), and the derivation based on the Fundamental matrix with continuation published by Luong and Faugeras (1997). The use of Singular Value Decomposition (SVD) of the Fundamental matrix to derive the Kruppa's equation were presented by Lourakis and Deriche (1999), Hartley (1997), Lourakis and Deriche (2000) and also discussed in the book of Hartley and Zisserman (2003, chap. 19). In addition, Pollefeys (1999) derived the Kruppa's equation from the homographies. Another paper, Sturm (2000) presented the study of the degenerate cases of the classical Kruppa's equation. Then, in later report of Ma et al. (2000), the study of particular degenerate case of Kruppa's approach based on the camera axis rotation was presented. Although many works have been done, efforts are still required to improve the algorithm especially to minimize the errors and the degeneracies.

Differ to those papers mentioned above, in this paper we discuss the classical Kruppa's equation and the simplified version of it using the SVD of the Fundamental matrix in the aim to improve the algorithm by firstly finding the inaccuracies of the derivation methods. In section 2, the theoretical background of the Kruppa's equation is described. Section 3 is the discussion about the inaccuracies found in the derivation techniques and then we concluded the work in Section 4.

2. The Kruppa's Equation

In this section, Kruppa's equation is derived as it was first described by Faugeras et al. (1992). Let's start with the absolute conic Ω, which satisfies

$$x_1^2 + x_2^2 + x_3^2 = x_4 = 0 \tag{1}$$

The Kruppa's equation makes use of the invariant property of Ω, where the images ω and ω' only depend on the intrinsic parameters of the camera. The tangent planes of Ω are also tangents to ω and ω' through epipolar transformation. Assuming that the point of intersection of the epipole $\mathbf{p}$ with the line at infinity is $[y_1, y_2, 0]$. l_1 is then an epipolar line $(\mathbf{p} \times \mathbf{y})$. Any point x is on the line if and only if $(\mathbf{p} \times \mathbf{y})^T x = 0$.

If D is the dual of ω then it satisfies $(\mathbf{p} \times \mathbf{y})^T D (\mathbf{p} \times \mathbf{y}) = 0$ and D is represented as a symmetric matrix, as described by Faugeras et al. (1992), as follows

$$D = \begin{bmatrix} -\delta_{23} & \delta_3 & \delta_2 \\ \delta_3 & -\delta_{13} & \delta_1 \\ \delta_2 & \delta_1 & -\delta_{12} \end{bmatrix}. \tag{2}$$

It has six independent parameters minus one for the scaling factor and only five parameters are remain to be recovered. By setting $y_3 = 0$ and substituting D with the equation 2, the equation now becomes

$$A_{11} y_1^2 + 2A_{12} y_1 y_2 + A_{22} y_2^2 = 0 \tag{3}$$

where

$$\begin{aligned} A_{11} &= -\delta_{13} p_3^2 - \delta_{12} p_2^2 - 2\delta_1 p_2 p_3 \\ A_{12} &= \delta_{12} p_1 p_2 - \delta_3 p_3^2 + \delta_2 p_2 p_3 + \delta_1 p_1 p_3 \\ A_{22} &= -\delta_{23} p_3^2 - \delta_{12} p_1^2 - 2\delta_2 p_1 p_3 \ . \end{aligned} \tag{4}$$

The same thing applied for the second view

$$A'_{11} {y'_1}^2 + 2A'_{12} y'_1 y'_2 + A'_{22} {y'_2}^2 = 0. \tag{5}$$

The pair l and l' is a line correspondence under homography H, if and only if, it satisfies $\mathbf{y}' = N\mathbf{y}$. Transformation from the line $y_3 = 0$ to $y'_3 = 0$ is done by the bilinear transformation N. For $\tau = \frac{y_2}{y_1}, \tau' = \frac{y'_2}{y'_1}$ then one obtains N which is equivalent to

$$\tau = \frac{a\tau + b}{c\tau + d}, \tag{6}$$

where a, b, c, d can be computed up to a scale factor from the two epipoles $\mathbf{p}$, $\mathbf{p}'$ and a set of point matches $q_i \leftrightarrow q_i$, $1 \leqslant i \leqslant n$. A linear least square procedure based on equation 5 with

$$\tau_i = \frac{p_3 q_{i3} - p_2 q_{i3}}{p_3 q_{i1} - p_1 q_{i3}} \qquad \tau'_i = \frac{p'_3 q'_{i3} - p'_2 q'_{i3}}{p'_3 q'_{i1} - p'_1 q'_{i3}} \ . \tag{7}$$

Using substitution, one then yields

$$
\begin{aligned}
A_{11}+2A_{12}\tau+A_{22}\tau^2=0 \\
A'_{11}(b\tau+c)^2+2A'_{12}(b\tau+c)(\tau+a)+ \\
A'_{22}(\tau+a)^2=0\,.
\end{aligned}
\tag{8}
$$

Each equation 8 is quadratic in τ and both are having the same roots. Kruppa's equation are obtained by using the cross ratio

$$
\begin{aligned}
\frac{A_{11}}{A'_{11}(b\tau+c)^2}=\frac{2A_{12}\tau}{2A'_{12}(b\tau+c)(\tau+a)}= \\
\frac{A_{22}\tau^2}{A'_{22}(\tau+a)^2}\,.
\end{aligned}
\tag{9}
$$

Therefore, three equation are needed to derive D, where only two are independent:

$$
\begin{aligned}
A_{12}(A'_{22}a^2+A'_{11}c^2+2A'_{12}ac)- \\
(A'_{12}c+A'_{22}a+A'_{11}bc+A'_{12}ab)A_{11}=0 \\
A_{22}(A'_{22}a^2+A'_{11}c^2+2A'_{12}ac)- \\
(2A'_{12}b+A'_{22}+A'_{11}b^2)A_{11}=0.
\end{aligned}
\tag{10}
$$

Let's move back to the equation 1. A point $\mathbf{X}$ is on the absolute conic if it satisfies $\mathbf{X}^T\Omega\mathbf{X}=0$. At this point, we can see that the absolute conic Ω forms an identity matrix I on the plane at infinity $x_4=0$. The equation now becomes $\mathbf{X}^T\mathbf{I}\mathbf{X}=0$, which is equal to $\mathbf{X}^T\mathbf{X}=0$. $\mathbf{X}$ is projected to the image plane as $\mathbf{x}=P\mathbf{X}$. Considering only the projection to the first image plane, where the rotation is the identity matrix and the camera is without being translated, then projection matrix is $P=K[I\mid 0]$ and the point $\mathbf{X}$ can be written as $(x,0)$. From this consideration, we can see that $P=K$. This means that P is invertible, because $|P|\neq 0$, and hence $x=P^{-1}\mathbf{x}$.

Recall the equation for the point on the absolute conic, by substitution, the equation now is $(P^{-1}\mathbf{x})^T(P^{-1}\mathbf{x})$ which is equal to $\mathbf{x}^TP^{-T}P^{-1}\mathbf{x}$. Replacing P with K gives $\mathbf{x}^TK^{-T}K^{-1}\mathbf{x}=0$. This is what exactly we are looking for. It is clearly seen that the equation for the point on the image of absolute conic is $\mathbf{x}^T\omega\mathbf{x}=0$ with $\omega=K^{-T}K^{-1}$. This leads to the conclusion that the image of absolute conic only depends on the intrinsic part of the camera K. Once the ω is recovered, one can derive K by the decomposition of ω. For easier computation, we can use the dual D of it which is the inverse of ω. D can be computed as KK^T. From this relation and referring to the form of D in equation 2, there are five conditions provided:

$$
\begin{aligned}
\delta_{13}\delta_{12}>0 \\
\delta_{23}\delta_{12}>0 \\
\delta_{13}\delta_{12}-\delta_1^2>0 \\
\delta_{23}\delta_{12}-\delta_2^2>0 \\
\frac{(\delta_3\delta_{12}+\delta_1\delta_2)^2}{(\delta_{13}\delta_{12}-\delta_1^2)(\delta_{23}\delta_{12}-\delta_2^2)}\leqslant 1
\end{aligned}
\tag{11}
$$

Those five constraints are required to hold, otherwise the intrinsic matrix will not be physically acceptable. Another condition is that if the angle between x and y axes is set to $\Theta=\pi/2$, then the last condition in equation 11 should be replaced with $\delta_3 = -\delta_1\delta_2/\delta_{12}$. Because only two unknowns can be determined by equation 10, it is suggested to work based on two camera motions, which can give two epipolar geometries. Therefore, another two pairs of epipolar lines can be obtained and with minimum of five lines, it is enough to recover all of the unknowns in D and then the intrinsic camera matrix can then be computed.

Another derivation of Kruppa's equation is explained below and the symbols are maintained as equation in the text source of Hartley and Zisserman (2003, p. 469).

Figure 1 shows how the Kruppa's relate to the image of the absolute conic. C and C' are the images of the absolute conic Cw lays in the plane at infinity π. Each pair of the epipolar lines l_1, l_2 and l'_1, l'_2 are tangent to C and C'. The epipolar plane that is tangent to the first image of absolute conic C is tangent to the absolute conic C_w and also tangent to the conic C'. Any point $\mathbf{x}$ lies on line $\mathbf{l}$ if it satisfies $\mathbf{l}^T\mathbf{x} = 0$. C^* and $C^{*\prime}$ are the dual of the images of the absolute conic C and C' respectively. From the first view, l_1 and l_2 are combined to

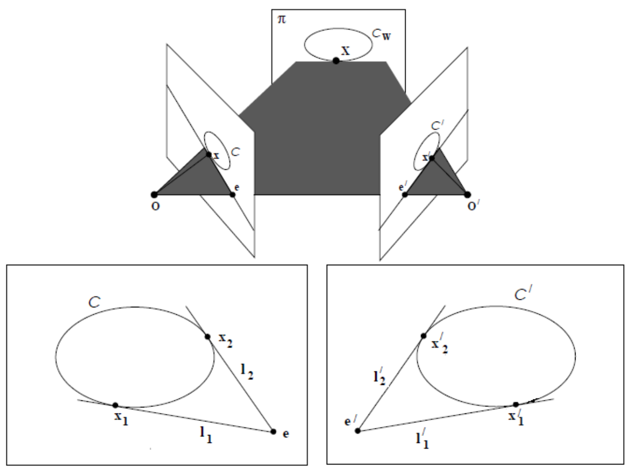

Fig. 1. Epipolar tangency for the conics.

define a single point conic $C_t = [e]_\times C^*[e]_\times$. Similarly, for the second view

$$C'_t = [e']_\times C^{*\prime}[e']_\times, \tag{12}$$

is given. Considering the homography H, the relation of both degenerated point conics can be written as $C'_t = H^{-T}C_tH^{-1}$ (Hartley and Zisserman, 2003, p. 37). By substitution, it yields

$$\begin{aligned} [e']_\times C^{*\prime}[e']_\times &= H^{-T}[e]_\times C^*[e]_\times H^{-1} \\ &= FC^*F^T \end{aligned} \tag{13}$$

and $C^* = \omega^*$, $C^{*\prime} = \omega^{*\prime}$, then the Kruppa's equation is defined by

$$[e']_\times \omega^{*\prime}[e']_\times = F\omega^*F^T \ . \tag{14}$$

From equation 14, the simplified Kruppa's equation is done by using the SVD of the Fundamental matrix. By considering that the Fundamental matrix has rank 2, the SVD of it is

$$F = UDV^T = U\begin{bmatrix} \sigma_1 & & \\ & \sigma_1 & \\ & & 0 \end{bmatrix}V^T \ . \tag{15}$$

From the above form, it can be seen that the corresponding vectors of the zero singular values are $\mathbf{u}_3$ and $\mathbf{v}_3$ and both vectors are the null-vectors of the Fundamental matrix. It means that the epipoles of the first and second view are $\mathbf{u}_3$ and $\mathbf{v}_3$, respectively. Replacing the epipoles on equation 14, results in

$$[\mathbf{u}_3]_\times \omega^{*\prime} [\mathbf{u}_3]_\times = UDV^T \omega^* VDU^T. \tag{16}$$

By using the property of U which is an orthogonal matrix,

$$U^T [\mathbf{u}_3]_\times \omega^{*\prime} [\mathbf{u}_3]_\times U = DV^T \omega^* VD. \tag{17}$$

Then both sides are reduced to symmetric matrices

$$\begin{bmatrix} u_2^T \omega^{*\prime} u_2 \\ -u_1^T \omega^{*\prime} u_2 \\ u_1^T \omega^{*\prime} u_1 \end{bmatrix} = \begin{bmatrix} \sigma_1^2 v_1^T \omega^* v_1 \\ \sigma_1 \sigma_2 v_1^T \omega^* v_2 \\ \sigma_2^2 v_2^T \omega^* v_2 \end{bmatrix}. \tag{18}$$

The solution is found by using the cross ratio

$$\frac{u_2^T \omega^{*\prime} u_2}{\sigma_1^2 v_1^T \omega^* v_1} = -\frac{u_1^T \omega^{*\prime} u_2}{\sigma_1 \sigma_2 v_1^T \omega^* v_2} = \frac{u_1^T \omega^{*\prime} u_1}{\sigma_2^2 v_2^T \omega^* v_2} \tag{19}$$

The intrinsic camera matrix K can be recovered by solving the dual image of the absolute conic ω^*. As long as ω^* is a symmetric and positive definite matrix, it then can be factorized $\omega^* = KK^T$ by using Cholesky factorization.

3. Inaccuracies of The Derivation

In homogeneous representation of a point in 2D Euclidean space, a conic equation can be written as $ax^2 + bxy + cy^2 + dxz + eyz + fz = 0$, where a, b, c, d, e, f are the coefficients with a, b, c not all zero. This quadratic equation can be represented as

$$\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0. \tag{20}$$

It means that a point $\mathbf{x}$ is on a conic C if and only if it satisfies $\mathbf{x}^T C \mathbf{x} = 0$, and C is a symmetric matrix. Then, the absolute conic Ω on equation 1 can be represented in matrix notation as

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0.$$

Considering that the plane at infinity is a 2D plane, if we define $x = \frac{x_1}{x_3}$ and $y = \frac{x_2}{x_3}$ where $x_4 = 0$, then the absolute conic is now can be written as $x^2 + y^2 = -1$ which represents an imaginary circle with radius of $\sqrt{-1}$. This imaginary circle, projected to the image plane by a projection P.

Let's recall the basic equation of the image of the absolute conic. A point $\mathbf{x}$ is on the image of the absolute conic if $\mathbf{x}^T \omega \mathbf{x} = 0$. Substituting $\mathbf{x} = P\mathbf{X}$ for the first camera by setting up $K = I$ so we have $P \neq 0$. This gives

$$\begin{aligned} (P\mathbf{X})^T \omega (P\mathbf{X}) &= 0 \\ \mathbf{X}^T P^T \omega P \mathbf{X} &= 0 \end{aligned}$$

and matrix of the absolute conic Ω is therefore $P^T \omega P$. In this case, P is an orthogonal matrix which means $\Omega = P^{-1}\omega P$. This makes Ω invariant under similarity transformation. From here we can see that the image of absolute conic, ω, is also an imaginary circle which lies on the image plane where all of the points are imaginary.

The dual of it, ω^*, is decomposed of imaginary lines. By looking at the equation of the imaginary circle, we can also define that only the coefficients of it can be real numbers. The first inaccuracy that can be described here is the points that are used by Faugeras et al. (1992) to derive the Kruppa's equation are the epipole $\mathbf{p}$ and a point at infinity $\mathbf{y}$. Those two points lead to the ambiguity on the generated tangent line $(\mathbf{p} \times \mathbf{y})$. The cross product of the two points $[p_1\ p_2\ p_3]^T \times [y_1\ y_2\ 0]^T$ forms a line

$$(-p_3 y_2, p_3 y_1, p_1 y_2 - p_2 y_1)^T. \tag{21}$$

If y_1 and y_2 are real numbers, then the coefficients of the line are real and if all the points that made up the line are real points, it gives a real line in the image plane and this contradicts the definition of the imaginary circle. The solution to this is to use the imaginary conjugate points, namely the circular points, on the plane at infinity $(1, i, 0)$ and $(1, -i, 0)$. The use of the circular points helps in generating an imaginary line when it is combined with the epipoles or any other real points on the plane. This can be seen that the cross product $[p_1\ p_2\ p_3]^T \times [1\ i\ 0]^T$, gives an imaginary line vector $(-p_3 i, p_3, p_1 i - p_2)^T$.

The derivation of Kruppa's equation in (Hartley and Zisserman, 2003, p. 469) which has already explained in section 2 as well, started from combining the two epipolar lines l_1 and l_2 into a degenerate point conic as shown in equation 12. The fact that matrix representing the conic in this case C, C' and $C*$, $C*'$ must be full rank matrices as required by Cholesky Decomposition method. A degenerate point conic matrix as derived in the equation 12 and 13 is not a full rank matrix, which means for a 3×3 matrix size the rank is < 3. If ω, ω' and $\omega*$, $\omega*'$ are treated in the same manner as explained in equation 14, it leads to the same answer where the matrix that is recovered will not be a full rank matrix. This also means that the matrix is not positive definite and it will not be possible to factorize it by Cholesky Decomposition method to recover the camera matrix.

From this point, we derived the Kruppa's equation in a similar manner to what was proposed in (Hartley and Zisserman, 2003, p. 469), by considering both epipolar lines on the two image planes and the relation of it to the dual of the image of the absolute conic.

Firstly, by considering that C^* is a dual conic which means the dual is a collection of lines. A line l is on the dual conic only if $l^T C^* l = 0$. Similarly, for the second view, $l'^T C^{*\prime} l' = 0$. By using the knowledge that a line l' is formed by the intersection of two points $l' = e' \times \mathbf{x}'$, and by using the skew symmetric matrix representation, the cross product can be written as $l' = [e']_\times \mathbf{x}'$. Then under homography H

$$\begin{aligned}\mathbf{x}'^T [e']_\times C^{*\prime} [e']_\times \mathbf{x}' &= \mathbf{x}'^T H^{-T} [e']_\times C^* [e']_\times H^{-1} \mathbf{x}' \\ &= \mathbf{x}'^T F C^* F^T \mathbf{x}'\end{aligned}$$

hence, $l^{T\prime} C^{*\prime} l' = l^T C^* l$. It is shown that both epipolar lines from the two image planes are equal in defining their dual. The Kruppa's equation now is

$$\mathbf{x}'^T [e']_\times C^{*\prime} [e']_\times \mathbf{x}' = \mathbf{x}'^T F C^* F^T \mathbf{x}'. \tag{22}$$

To find the suitable $\mathbf{x}'$ which leads to the generation of imaginary tangent lines, we can now make use the circular points at infinity.

Using the simplified Kruppa's equation derived from the SVD of the Fundamental matrix as in the equation 19, leads to uncovered $\omega^{*\prime}$. It can be seen as follows. Assuming the projection matrix for the first camera where the translation matrix is $\mathbf{t} = 0$ and for the second camera the translation is $\mathbf{t}' = [\lambda\ 0\ 0]^T$. There is no rotation between camera giving the rotation matrix as $R = R' = I$. Both camera maintain the intrinsic matrix $K = K' = I$. From those knowledge, the first camera matrix is $P = [I \mid 0]$ and the second camera matrix is $P' = [I \mid \mathbf{t}']$. The epipole in the second view and the Fundamental matrix of both views are defined by Hartley and Zisserman (2003, p. 244)

$$
\begin{aligned}
e' &= K'\mathbf{t} \\
F &= [e']_\times P'P^+ \ ,
\end{aligned}
$$

computing e' and F give

$$
e' = \begin{bmatrix} \lambda \\ 0 \\ 0 \end{bmatrix} \quad F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\lambda \\ 0 & \lambda & 0 \end{bmatrix} \ ,
$$

which results in

$$
\begin{aligned}
\mathrm{SVD}(F) = U\ D\ V^T = \\
\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}^T
\end{aligned}
$$

The degeneracy occurs if the values of u_1, u_2 and v_1, v_2 of equation 19 are substituted with the above results. The zero values on the columns of U and V lead to a zero result for the equation and the unknowns of ω^* can not be resolved. This is agreed the discussion in (Hartley and Zisserman, 2003, p. 473) where it is mentioned if there is no rotation between the two cameras then the Kruppa's provides no constraint on ω^*.

In general, the condition that the Fundamental matrix is forced to have a norm $\|\mathbf{f}\| = 1$, gives rise to very small output which then be fed as the input to the Kruppa's equation. This can cause a numerical error. One possible solution for this is by multiplying the values of the vectors U and V of the SVD of the Fundamental matrix with a scalar number c become $U = cU$ and $V = cV$, where $c \in \mathbb{R} \setminus \{0, 1\}$, to prevent the zero result of the conic matrix as the solution of the Kruppa's equation.

4. Conclusion

The Kruppa's methods for camera self-calibration were reviewed and there are some points concluded as follows:

First, the method which was introduced by Faugeras et al. (1992) gives real tangent lines which are generated from the epipoles and any points at infinity. It leads to a false derivation of an imaginary conic. By making use the epipole and the the circular points at infinity to generate the tangent lines, will make sure that the image of the absolute conic and the dual of it, will be imaginary. The equation can only solve two unknowns out of three unknowns in the matrix of imaginary circle. One of the solutions is to take another image as the input and using the two epipolar transformations from those three images, all the unknowns can be recovered. Second, similar inaccuracies are also present in the Kruppa's equation as shown in equation 13. The consequence of using of the degenerate point conic as a starting point to relate the equation with the Fundamental matrix is, it will never meet the goal to generate matrix of an imaginary circle. Other than that, the inaccuracy by making use the Fundamental matrix which is in the fact, forced to have the matrix norm $\|\mathbf{f}\| = 1$. Therefore each element of $\mathbf{f}$ is $\leqslant 1$ and this increases the difficulty of finding the real values of the intrinsic matrix K. Similar thing happens for the simplified version of the Kruppa's equation which is derived from the SVD of the Fundamental matrix. The SVD method uses the unit vector for the values of the two orthonormal vectors U and V. Hence, the values of the real intrinsic matrix will never be recovered. Third, if the work is based on the imaginary circle which is projected from the absolute conic, the constraints of a circle (eq. 20) that need to be considered are $a = c$ and $b = 0$. From this point, in the relation to the intrinsic parameters K, the constraints should be restricted from the beginning: scaling factors α_x and α_y, both should have the same values. It means the camera pixels are square and focal lengths in the direction x axes and y axes are the same. Furthermore, the skew parameter s should be set to zero.

It should be noticed that the absolute conic alone is a special conic when it is considered to have only the two circular points at infinity lie on it. Those circular points act as the vertices of the conic and the locus is the line at infinity taken twice (Semple and Kneebone, 1998, p. 121). The absolute conic itself, in this case, is a degenerate conic. If those points or tangents are simply projected to the image plane as another degenerate conic, then the rank of degenerate conic is not a full rank. This means that the matrix is not positive definite. From this fact we concluded that, in practice, Cholesky Decomposition will never work to decompose the conic matrix into intrinsic camera matrix, as it needs an input which is a symmetric and positive definite matrix. Unless, the matrix is first normalized to match the constraints of Cholesky decomposition method.

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