Proceedings of the World Congress on New Technologies (NewTech 2015)
Barcelona, Spain – July 15 - 17, 2015
Paper No. 313

Free Vibration Analysis of a Cracked Atomic Force Microscope Cantilever

Win-Jin Chang, Haw-Long Lee, Yu-Ching Yang
Department of Mechanical Engineering, Kun Shan University, Taiwan
changwj@mail.ksu.edu.tw

Abstract **-**In this paper, the vibration response of an atomic force microscope (AFM) cantilever with a crack is analyzed using a modified couple stress theory. The effects of crack location and contact stiffness on the modal sensitivity of the cracked AFM cantilever are investigated. According to the analysis, the crack is closed to the fixed end of the cantilever that has a higher modal sensitivity. In addition, the modal sensitivity of the cantilever using the modified couple stress theory is smaller than that using the classical beam theory for the lower contact stiffness. However, the situation is reversed when the contact stiffness becomes larger.

Keywords**:** Atomic force microscope; Modal sensitivity; Cracked cantilever; Modified couple stress theory.

1. Introduction

An atomic force microscope (AFM) has been widely used for surface imaging of conductors and insulators on an atomic scale, see for example, Holmberg and Matthews (1994), Dirote (2004), and Gehar (2006). The AFM has a flexible cantilever with a sharp pyramidal or conical tip at its free end and is used to scan a sample surface in contact mode. It is well known that the cantilever plays an important role in AFM measurements. When the tip scans across the sample surface, the cantilever induces a dynamic interaction force between the tip and the surface.

The resolution of measurement for the AFM cantilever is related to its vibration sensitivity. In the last few years, many researchers have much interest in studying the resonant frequency and sensitivity analysis of AFM cantilevers. The modified couple stress theory, which was proposed by Yang et al. (2002), can also be used to estimate the vibration behaviors of an AFM cantilever. This theory was modified from the classical couples stress theory and has been well established since the early 1960s. Unlike the classical continuum theory, the modified couple stress theory includes one additional material length scale parameter revealing the micro-scale effect on the response of structures to estimate the size-dependent behaviours. In the recent years, some researchers utilized the modified couple stress theory to study the vibration sensitivity of an AFM cantilever. For example, Kahrobaiyan et al. (2010) investigated the resonant frequency and sensitivity of AFM rectangular cantilevers using the modified couple stress theory. Lee and Chang (2011) analyzed the sensitivity of the flexural vibration modes for a V-shaped atomic force microscope cantilever using the Rayleigh-Ritz method.

Cracks may be induced in an AFM cantilever during the fabrication process, see for example, Shafai *et al*. (1998) and Joshi *et al.* (2010). Cracks will affect the sensitivity of AFM cantilevers during scanning. However, there are very few literature reports on the crack effect of AFM cantilevers. Lee and Chang (2012a, 2012b) analyzed the sensitivity and dynamic response of an AFM cantilever with a crack based on the classical continuum theory. In this paper, the modified couple stress theory is used to predict the effects of crack location, and contact stiffness on the modal sensitivity of a cracked cantilever. In addition, the comparison of the results obtained by the classical and modified couple stress theories is also presented.

2. Vibration Analysis

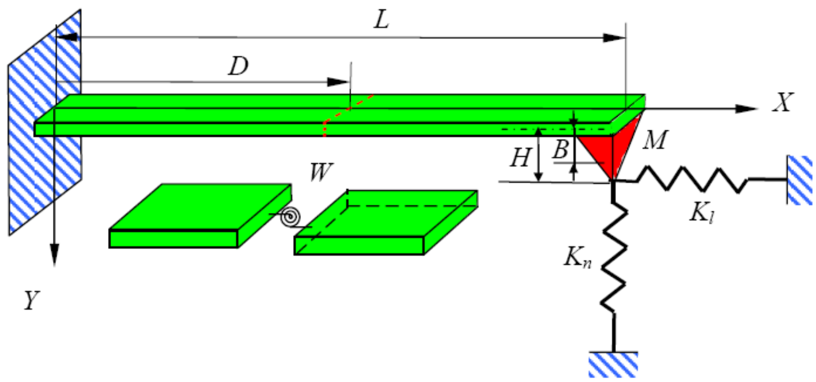

Fig. 1. Schematic diagram of a cracked cantilever of an AFM in contact with a sample.

An AFM probe is considered as a cantilever beam which have the density ρ, Poisson's ratio μ, uniform cross-section *A*, thickness *h*, and length *L*. Assume the probe has a crack at a distance *D* from the fixed end as shown in Fig. 1. Therefore, the cantilever will be divided into two segments by the crack. A rigidity of the rotational spring simulated the cracked section to connect the two segments of the cantilever. The governing equation of transverse vibration of the cantilever with a crack can be expressed as

$$(EI+GAl^2)\frac{\partial^4 Y_1}{\partial X^4}+\rho A\frac{\partial^2 Y_1}{\partial t^2}=0,\quad 0\le X\le D \tag{1}$$

$$(EI+GAl^2)\frac{\partial^4 Y_2}{\partial X^4}+\rho A\frac{\partial^2 Y_2}{\partial t^2}=0,\quad D\le X\le L \tag{2}$$

where *E* and *G* are the Young's modulus and shear modulus, respectively; *I* is the area moment of inertia; *l* is the material length scale parameter which indicates the size-dependent behavior of the microcantilever based on the modified coupled stress theory; *X* is the distance from the fixed end along the center of the probe, *t* is time, $Y_1(X,t)$ and $Y_2(X,t)$ is the transverse displacement of both segments, respectively.

For compatibility of displacement, moment and shear force for the two adjacent portions of the cantilever can be expressed by the following jump conditions as

$$Y_1(D,t)=Y_2(D,t) \tag{3}$$

$$\frac{\partial^2 Y_1(D,t)}{\partial X^2}=\frac{\partial^2 Y_2(D,t)}{\partial X^2} \tag{4}$$

$$\frac{\partial^3 Y_1(D,t)}{\partial X^3}=\frac{\partial^3 Y_2(D,t)}{\partial X^3} \tag{5}$$

The crack is simulated by a rotational spring and the angular displacement between the two segments can be related by

$$W\left(\frac{\partial Y_2(D,t)}{\partial X} - \frac{\partial Y_1(D,t)}{\partial X}\right) = (EI + GAl^2)\frac{\partial^2 Y_1(D,t)}{\partial X^2} \tag{6}$$

Where W is the rotational spring constant.

In addition, the boundary conditions of two ends of the AFM cantilever are

$$Y_1(0,t) = 0 \tag{7}$$

$$\frac{\partial Y_1(0,t)}{\partial X} = 0 \tag{8}$$

$$(EI + GAl^2)\frac{\partial^2 Y_2(L,t)}{\partial X^2} = -H^2 K_l \frac{\partial Y_2(L,t)}{\partial X} - MB^2 \frac{\partial^3 Y_2(L,t)}{\partial X \partial t^2} \tag{9}$$

$$(EI + GAl^2)\frac{\partial^3 Y_2(L,t)}{\partial X^3} = K_n Y_2(L,t) + M\frac{\partial^2 Y_2(L,t)}{\partial t^2} \tag{10}$$

Where H and M are the tip height and mass, respectively. B is the distance between the lower edge of the probe and centroid of the cross section. The boundary condition of the cantilever is assumed to be a fixed end at $X = 0$; then the boundary conditions given by Eqs. (7) and (8) correspond to the conditions of zero displacement and zero slope. Furthermore, the Eqs. (9) and (10) correspond to the moment at $X = L$ and the force balanced between the cantilever and a linear tip–sample interaction, respectively. The linear spring constants K_n and K_l denote the contact stiffness between the tip and the sample under the normal and lateral direction, respectively. The first term of right side in Eq. (9) represents the moment due to the lateral tip-sample interaction force and the second term denotes the moment due to the mass moment of inertia of the tip. Similarly, the first term of the right side in Eq. (10) is the normal tip-sample interaction force and the second term is the inertia force of tip mass.

We seek harmonic solution of the form as

$$Y_1(X,t) = v_1(X)e^{i\omega t}, \quad Y_2(X,t) = v_2(X)e^{i\omega t} \tag{11}$$

Where ω is the radian frequency.

The dimensionless variables are defined as

$$y_1 = \frac{v_1}{L}, \quad y_1 = \frac{v_1}{L}, \quad m = \frac{M}{\rho AL}, \quad z = \frac{H}{L}, \quad \delta = \frac{B}{L}, \quad \beta_n = \frac{K_n L^3}{EI}, \quad \beta_l = \frac{K_l L^3}{EI}, \quad \gamma = \sqrt[4]{\frac{\rho AL^4}{EI}\omega^2}, \quad k_c = \frac{EI}{WL},$$
$$\alpha = \frac{D}{L}, \quad \eta = \frac{GAl^2}{EI} = \frac{12G}{E\left(h/l\right)^2}. \tag{12}$$

Where y_1 and y_2 are the dimensionless transverse displacement of the two segments. m, z and δ are the dimensionless tip mass, tip height and its centroid. β_n, β_l and γ are the dimensionless normal, lateral contact stiffness and wave number. k_c and α are the dimensionless flexibility and location of the crack, respectively. Meanwhile, η is the dimensionless effective flexural rigidity based on the modified coupled stress theory, where h/l is the ratio of the cantilever thickness to material length scale parameter. After a lengthy calculation, the frequency equation for the AFM cantilever with a crack is obtained as follows:

$$F(\gamma,\beta_n)=\begin{vmatrix} s & c & sh & ch & -s & -c & -sh & -ch \\ \lambda k_c s-\eta_1 c & \lambda k_c c+\eta_1 s & -\eta_1 ch-\lambda k_c sh & -\lambda k_c ch-\eta_1 sh & \eta_1 c & -\eta_1 s & \eta_1 ch & \eta_1 sh \\ s & c & -sh & -ch & -s & -c & sh & ch \\ c & -s & -ch & -sh & -c & s & ch & sh \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \xi C-\eta_1\lambda S & -\xi S-\eta_1\lambda C & \xi Ch+\eta_1\lambda Sh & \xi Sh+\eta_1\lambda Ch \\ 0 & 0 & 0 & 0 & \zeta S-\lambda^3 C & \zeta C+\lambda^3 S & \zeta Sh+\lambda^3 Ch & \zeta Ch+\lambda^3 Sh \end{vmatrix}=0 \quad (13)$$

Where

$$c=\cos(\alpha\lambda),\ \ s=\sin(\alpha\lambda),\ ch=\cosh(\alpha\lambda),\ \ sh=\sinh(\alpha\lambda),\qquad C=\cos(\lambda),\ \ S=\sin(\lambda),\ Ch=\cosh(\lambda),\ \ Sh=\sinh(\lambda)$$
$$\xi=\beta_l z^2-m\delta^2\eta_1\lambda^4\ ,\quad \zeta=m\lambda^4-\beta_n/\eta_1\ ,\ \text{and}\ \ \eta_1=1+\eta\ . \quad (14)$$

In addition, the dimensionless modal sensitivity of the cracked AFM cantilever based on the modified couple stress theory can be expressed as

$$S_n=\frac{dF/d\beta_n}{\frac{1}{2\pi}\sqrt{\frac{EI}{\rho AL^4}}}=-2\lambda\sqrt{1+\eta}\,\frac{\partial F/\partial\beta_n}{\partial F/\partial\lambda} \quad (15)$$

3. Results and Discussion

This article is to study the effects of crack location and contact stiffness on the sensitivity of the vibration modes of a cracked AFM cantilever based on the modified couple stress theory. The modal sensitivity is defined as the change in the flexural frequency of the mode to the change in the tip-sample interaction. In order to know the effect of relative parameters on the sensitivity of a cracked AFM cantilever, we considered the geometric and material parameters of a Si cantilever as follows: E = 170 GPa, μ =0.28, $\rho=2330\,kg/m^3$, $L=300\mu m$, B = 2.5 μm, H = 10 μm, M = $2\times10^{-13}kg$. The lateral contact stiffness was assumed as $K_l=0.9K_n$.

Figure 2 shows the comparison of sensitivity of different modes obtained by the classical and modified couple stress theories for a cracked cantilever of an AFM with H/L =1/30, D/L=0.8, k_c = 0.4, and h/l=4. Unlike the classical beam theory, the modified couple stress theory includes the effect of small-scale structure and that leads to a stiffer cantilever. Therefore, when the contact stiffness is small, it can be seen that the sensitivity of the AFM cantilever using the modified couple stress theory is smaller than that using the classical beam theory which is obtained by Lee and Chang (2012a). The situation is reversed when the contact stiffness becomes large. This indicates that a softer cantilever is easier to bent and more sensitive for imaging of soft biological samples and a stiffer cantilever is preferred for scanning a harder surface.

Figure 3 illustrates the dimensionless sensitivity of mode 1 is functions of the microcantilever thickness to material length scale parameter *h/l* and dimensionless crack location D/L for H/L =1/30, k_c = 0.4, and $\beta_n=0.1$. The elastic energy of a cracked cantilever during scanning obviously decreases as the crack is close to the fixed end. This implies that the crack away from the free end shows significant changes in the frequency. Therefore, the sensitivity is high for a lower value of *D/L*. In addition, it can be seen that the sensitivity of mode 1 obviously increases with an increase of *h/l* and with a decrease of *D/L*. This is useful for the design of a high-sensitivity AFM cantilever.

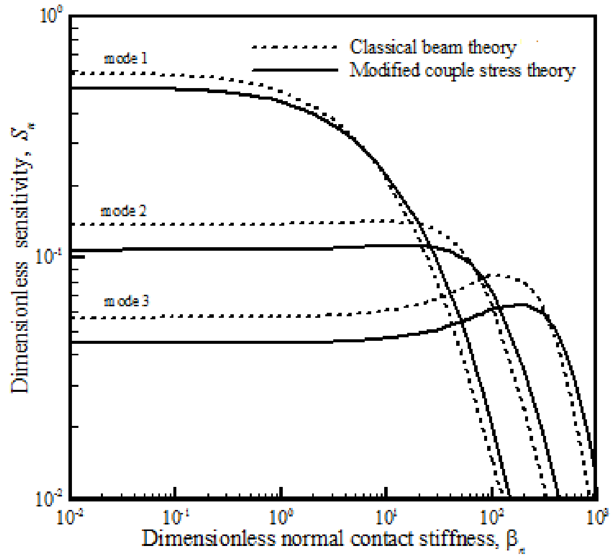

Fig. 2. Comparison of sensitivity obtained by the classical and modified couple stress theory for a cracked cantilever of an AFM with H/L =1/30, D/L=0.8, k_c = 0.4, and h/l=4.

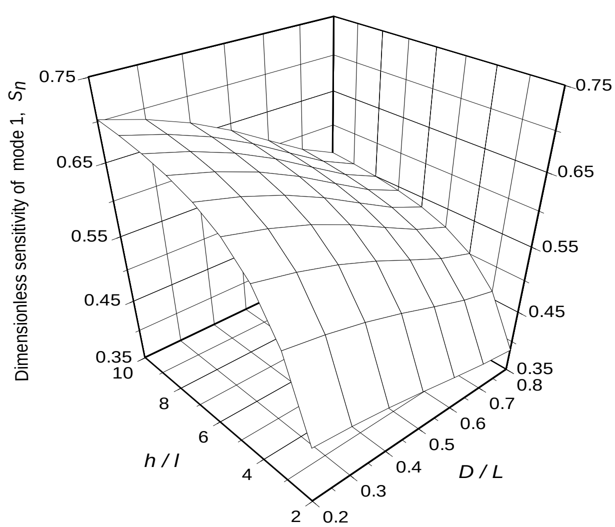

Fig. 3. Dimensionless sensitivity of mode 1 is functions of the microcantilever thickness to material length scale parameter *h/l* and dimensionless crack location D/L for H/L =1/30, k_c = 0.4, and $\beta_n = 0.1$.

4. **Conclusion**

The effect of crack location on the sensitivity of a cracked AFM cantilever has been analyzed based on the modified couple stress theory. An explicit expression for the frequency and sensitivity of vibration modes of the cracked cantilever was obtained using the relationship between the resonant frequency and tip-sample interaction. Results showed that a lower sensitivity was obtained as the thickness of cantilever was close to the material length scale parameter. In addition, the crack was closed to the fixed end of the cantilever that had a higher modal sensitivity.

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