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Analysis of Parallel Plate Waveguide with Anisotropic Chiral Medium

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Abstract -Theoretical study of electromagnetic wave propagation in parallel plate waveguide with chiral medium is presented. The medium understudy is considered as axial anisotropic case, where the tensors of permittivity, permeability and chirality parameters are diagonal. This research work is based on the algebraic formulation of Maxwell equations according to the A formalism of constitutive relations. The dispersion modal equations is obtained and solved, the results of these equations confirmed the specificity of the bifurcation modes, subject of this study. Three regions are identified: the fast-fast wave region, the fast-slow wave region, and the slow-slow wave region. The paper is basically focused on the first region, the effect of chirality parameter (ξ), is considered, as well as propriety of modal bifurcation in K_+ and K_-.

***Keywords*:** anisotropic, chiral medium, parallel-plate, chirowaveguide.

1. Introduction

Bi-anisotropic media have interesting applications in electromagnetic wave propagation and, their main problem lies in their manufacturing which has not reached the level of industrialization yet, however, before starting making such media, their usefulness must be shown theoretically. A bi anisotropic medium is specified by the following constitutive equations: as shown by Sihvola A. (1993)

$$\mathbf{B} = [\mu]\mathbf{H} + ([\chi] + j[\xi])\sqrt{\mu_0\varepsilon_0}\,\mathbf{E} \quad (1)$$

$$\mathbf{D} = [\varepsilon]\mathbf{E} + ([\chi] - j[\xi])\sqrt{\mu_0\varepsilon_0}\,\mathbf{H} \quad (2)$$

Where **E**, **H**, **D** and **B** represent respectively the electric field, the magnetic field, the electric flux density, and the magnetic flux density, $[\varepsilon]$, $[\mu]$, $[\chi]$ and $[\xi]$ are the physical parameters of the material, which define the proprieties of the medium. The absolute values of $[\chi]$ and $[\xi]$ are within zero and one as shown by Edward Ji. (2002). Furthermore, it is useful to mention the different classifications of a given medium, depending on the chirality parameter and the non-reciprocity as shown in Tab.1, given by Zarifi D.(2014). Indeed, chiral media make a subset of bi-anisotropic media cases. Therefore, this study will be based on Pasteur medium which is reciprocal chiral anisotropic, i.e $([\chi]=0)$ and $([\xi] \neq 0)$.

In this work, an interest is given to wave propagation study in chiral-core plane waveguide according to its different physical parameters diversity. Waves propagation in anisotropic chiral medium are modeled and studied (where tensors of chirality, permittivity and permeability are diagonal), in which the A formalism is used as shown by Ougier S. (1994), related to the choice of the proposed structure, and that will enable the lightening of analytical calculation procedure of Maxwell equations. First of all, the first region is treated, where the cutoff frequencies of each case are determined, and the curves of normalized propagation constants according to normalized frequencies are drawn.

Table. 1. Medium classification with respect to chirality and non reciprocity parameters.

	nonchiral medium $([\xi]=0)$	chiral medium $([\xi]\neq 0)$
reciprocal medium $([\chi]=0)$	simple isotropic (or anisotropic) medium	chiral medium (or pasteur)
non reciprocal medium $([\chi]\neq 0)$	Tellegen medium	general bi-isotropic (or bi-anisotropic) medium

2. Theory

2. 1. Problem Formulation

In this section we analyzed the chiral parallel plate wave guide depicted in fig. 1, with perfectly conducting planes placed at $x=\pm a$. Chirowaveguides propagation direction is along z axis. The field quantities are all independent of y axis.

The permittivity, the permeability and the chirality tensors of the adopted environment are:

$$[\varepsilon]=\mathrm{diag}[\varepsilon_x,\varepsilon_y,\varepsilon_z]\ ,\ [\mu]=\mathrm{diag}[\mu_x,\mu_y,\mu_z]\ \text{and}\ [\xi]=\mathrm{diag}[\xi_x,\xi_y,\xi_z] \tag{3}$$

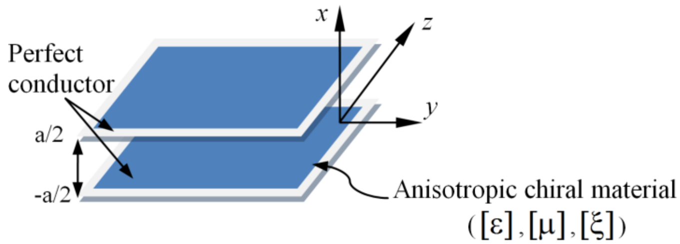

Fig. 1. A parallel-plate waveguide composed with perfect conductor, filled with an anisotropic chiral material

After algebraic manipulation of Maxwell's equations, we obtain the following set of equations

$$\frac{\partial^2}{\partial x^2}\begin{Bmatrix}E_z\\H_z\end{Bmatrix}+\left(\left(\omega^2\mu_z\varepsilon_z+\beta_0^2\xi_z^2\right)-\beta^2\right)\begin{Bmatrix}E_z\\H_z\end{Bmatrix}\mp j2\omega^2\sqrt{\mu_0\varepsilon_0}\,\xi_z\begin{Bmatrix}\mu_z H_z\\\varepsilon_z E_z\end{Bmatrix}=0. \tag{4}$$

The differential coupled equations of the electric E and magnetic H field along the 'oz' axis are presented by Eqs. (4), the following equations are necessary to decouple the previous pair of equations.

$$H_z^{'}=\sqrt{\tfrac{\mu_z}{\varepsilon_z}}H_z \tag{5}$$

$$(E_z+jH_z^{'})=A_z \tag{6 a}$$

$$(E_z-jH_z^{'})=B_z \tag{6 b}$$

$$K_{+i}=\left(\omega\sqrt{\mu_i\varepsilon_i}-\beta_0\xi_i\right)\ \text{and}\ K_{-i}=\left(\omega\sqrt{\mu_i\varepsilon_i}+\beta_0\xi_i\right)\ \text{with i=x,y and z} \tag{7}$$

We deduced the following equations:

$$\frac{\partial^2}{\partial x^2}A_z+\left(K_{+z}^2-\beta^2\right)A_z=0 \tag{8 a}$$

$$\frac{\partial^2}{\partial x^2}B_z+\left(K_{-z}^2-\beta^2\right)B_z=0 \tag{8 b}$$

Before starting the resolution of the differential equations It is important, to take into account the interesting propriety imposed by the chiral medium which are illustrated by the following cases, represented in Tab .2, as shown by Pellet P.(1990).

Table. 2. Cases classification depending on velocities of RCP, LCP and guided velocity.

Cases	conditions	Velocities	Type of solutions
fast-fast wave region. $U_{1,2}=\sqrt{\left(K_{\pm z}^2-\beta^2\right)}$	$\beta<K_{-z}<K_{+z}$ and $\xi_z<0$	$v_{gz}>v_{RCP}>v_{LCP}$	$\begin{cases}A_z=A_1\cos U_1x+A_2\sin U_1x\\ B_z=B_1\cos U_2x+B_2\sin U_2x\end{cases}$
fast-slow wave region. $U_1=\sqrt{\left(K_{+z}^2-\beta^2\right)}$ $U_2=\sqrt{\left(\beta^2-K_{-z}^2\right)}$	$K_{-z}<\beta<K_{+z}$ and $\xi_z<0$	$v_{LCP}>v_{gz}>v_{RCP}$	$\begin{cases}A_z=A_1\cos U_1x+A_2\sin U_1x\\ B_z=B_1\cosh U_2x+B_2\sinh U_2x\end{cases}$
slow-slow wave region $U_{1,2}=\sqrt{\left(\beta^2-K_{\pm z}^2\right)}$	$K_{-z}<K_{+z}<\beta$ and $\xi_z<0$	$v_{LCP}>v_{RCP}>v_{gz}$	$\begin{cases}A_z=A_1\cosh U_1x+A_2\sinh U_1x\\ B_z=B_1\cosh U_2x+B_2\sinh U_2x\end{cases}$

2. 2. Fast-Fast Wave Region

The propagation constant, of the propagating modes, is a positive real number. And the specific boundary conditions imposed by the adopted structure, lead to the tangential components E equal to zero at the surface of the metal walls, as shown by, Topa A. L.(2010), and Ghafar A. (2014). So, the application of these boundary conditions allows deducing the following matrix form:

$$[A]=\begin{bmatrix}\cos\frac{U_1a}{2} & \sin\frac{U_1a}{2} & \cos\frac{U_2a}{2} & \sin\frac{U_2a}{2}\\ \cos\frac{U_1a}{2} & -\sin\frac{U_1a}{2} & \cos\frac{U_2a}{2} & -\sin\frac{U_2a}{2}\\ -f_1\sin\frac{U_1a}{2} & f_1\cos\frac{U_1a}{2} & +f_2\sin\frac{U_2a}{2} & -f_2\cos\frac{U_2a}{2}\\ f_1\sin\frac{U_1a}{2} & f_1\cos\frac{U_1a}{2} & -f_2\sin\frac{U_2a}{2} & -f_2\cos\frac{U_2a}{2}\end{bmatrix} \tag{9}$$

Nontrivial solutions are obtained when det[A]=0, which leades to the following:

$$\Delta_{1,2}=\pm\left(\frac{f_1}{2}+\frac{f_2}{2}\right)\sin\left(\frac{U_1a}{2}+\frac{U_2a}{2}\right)+\left(\frac{f_1}{2}-\frac{f_2}{2}\right)\sin\left(\frac{U_1a}{2}-\frac{U_2a}{2}\right)=0 \tag{10}$$

$$f_i=\frac{k_{\pm x}U_i}{(k_{\pm x}k_{\pm y}-\beta^2)}\ \text{with i=1,2} \tag{11}$$

Δ_1 and Δ_2 represent the dispersion relations in the parallel-plate of the chirowaveguide. Before solving the dispersion relation, cutoff frequencies have to be calculated. According to the chirality parameter there are three cases:

1- If $\xi_z\sqrt{\mu_{ry}\varepsilon_{ry}} = \xi_y\sqrt{\mu_{rz}\varepsilon_{rz}}$ and $\sqrt{\mu_{ry}\varepsilon_{ry}} \neq |\xi_y|$ (12)

We have $f_c = \dfrac{n}{2\sqrt{\mu_z\varepsilon_z}a}$ (13)

2- If $\sqrt{\mu_{ry}\varepsilon_{ry}}\sqrt{\mu_{rz}\varepsilon_z} = \xi_z\xi_y$ and $\sqrt{\mu_{ry}\varepsilon_{ry}} \neq |\xi_y|$ (14)

We have $f_c = \dfrac{n}{2\sqrt{\varepsilon_0\mu_0}\xi_z a}$ (15)

In this second case, and according to this interesting formula (Eq. 15) deduced from our calculations, chirality removes the direct effect of two parameters (permeability and permittivity) on the cutoff frequency. The chiral parameter will remain the only influence. Therefore, it is easy to have low chiral parameters generating a very high cutoff frequency, and leading to important and interesting results that can be used to design high pass filters. This is in agreement with existing researches in the literature treating improvement of high pass filters by chiral metamaterials, as shown by Sabah C. (2012).

3-If $\begin{cases} \xi_z\sqrt{\mu_{ry}\varepsilon_{ry}} \neq \xi_y\sqrt{\mu_{rz}\varepsilon_z} \\ \sqrt{\mu_{ry}\varepsilon_{ry}}\sqrt{\mu_{rz}\varepsilon_z} \neq \xi_z\xi_y \\ \sqrt{\mu_{ry}\varepsilon_{ry}} \neq |\xi_y| \end{cases}$ (16)

We have $f_c = \dfrac{n}{2\sqrt{\mu_z\varepsilon_z}a} = \dfrac{n'}{2\sqrt{\varepsilon_0\mu_0}\xi_z a}$ (17)

Where $\dfrac{n}{n'} = \dfrac{\sqrt{\mu_{rz}\varepsilon_{rz}}}{\xi_z}$, Thus $\xi_z = \dfrac{n'}{n}\sqrt{\mu_{rz}\varepsilon_{rz}} \Rightarrow \xi_z = \dfrac{n' * m}{n * m}\sqrt{\mu_{rz}\varepsilon_{rz}}$ (18)

And m: indicates the number of modes for each value of n and n'

3. Conclusion

In this paper parallel plate waveguide containing homogeneous anisotropic chiral material has been studied and analyzed. The dispersion relation, Brillouin diagrams, cutoff frequencies, and propagating modes for the waveguide loaded with chiral medium have been obtained. It has been shown that the propagating modes are bifurcated. The particular cases deduced from this generalized study are based on a simple formulation of the chiral-core waveguide using physical parameters This latter could be either anisotropic (or isotropic) chiral, or anisotropic (or isotropic) achiral, which presents the originality of this work. The novelty of this study is based on a generalized calculation, where the studied media are anisotropic chiral according to the axial case, this enables to treat an important diversity of structures in different materials such as: chiral metamaterials, plasma, magnetic materials, and dielectrics according to their physical parameter values.

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