Proceedings of the 3rd World Congress on New Technologies (NewTech'17)
Rome, Italy – June 6 – 8, 2017
Paper No. ICEPR 132
ISSN: 2369-8128
DOI: 10.11159/icepr17.132

Reliability and Uncertainty in Analysis of Rammed Earth Walls

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Abstract – This paper deals with reliability analysis and uncertainty of rammed earth (RE) structures as a widely used sustainable and environment-friendly structure. Due to lack of comprehensive design standards, the engineers often rely on "rule-of-thumb" methods, which leads to quite conservative or unsafe designs. In this study, uncertainty of load and resistance parameters were included in analysis of rammed earth structures. The reliability index and failure probability of RE structures were evaluated using First-Order-Reliability-Method (FORM). The analysis was performed based on the different loads and resistance random variables parameters. Based on the results, the recommended wall thickness by various codes are quite conservative. On the other hands, larger wall thickness is required under severe loadings conditions. The compressive strength of unstabilized materials under severe loading conditions should be more than minimum recommended. The sensitivity analysis on the random variables indicates that the compressive strength and the environmental loads factors are the most important random variables that contribute to reliability of the structures.

***Keywords*:** Rammed Earth; Sustainable Structure; Reliability; Earthen Materials; FORM; Sensitivity Analysis

1. Introduction

Rammed earth as a construction method has become widespread these days, due to its advantages such as low construction cost, low embodied energy and recyclability of materials. The required materials are natural, cheap, tough, and green. This method is helpful when using modern technologies is not possible, and where skilled labors are not available, or due to impassable roads the cost of transportation is relatively high. However, as they are susceptible to erosion and physical damages, they need continuous maintenance. Their high mass, low ductility and low tensile and shear strength lead to detrimental damages during high or moderate ground motions [1-4].

The rammed earth structures are typically single or two-story buildings, supported by bearing walls. The appropriate mixture of soil is prepared in near its optimum moisture content to maximize its dry density and is formed in temporary formworks and compacted in layers by using manual or pneumatic rammers. Rammed earth structures are divided to two groups: Stabilized and unstabilized rammed earth. When the binder is just clay, this material is referred as "Unstabilized Rammed Earth" (URE). "Stabilized Rammed Earth" (SRE) is used when the composition is stabilized by cement or lime to enhanced mechanical properties and durability of the structure. Stabilization, however, has negative environmental effects such as increased embodied energy [1, 5, 6].

Up to now, there are limited comprehensive design and construction codes for RE structures compared to steel and concrete structures. There available guidelines and handbooks give mostly some recommendations and advices in this regard. This is because of variety of soil compositions and not sufficient researches on the behavior of rammed earth structures. Hence, the design of rammed earth structures are traditionally based on engineering judgement and experiences that generally lead to conservative designs. On the other hands, unsafe design is predicted of the structures subjected to severe environmental loads such as seismic ground motions, heavy snow loads and storms [6-14].

The objective of this research is to consider the uncertainty of parameters affecting the performance of URE and cement stabilized rammed earth (CSRE) structures under common environmental loads. The results of reliability analysis of rammed earth structures are used to check the validity of the recommendations on the wall thickness and compressive strength of materials. The parameters with have most effect on the probabilistic analysis of buildings are determined by sensitivity analysis.

2. Resistance Random Variables

Almost all the parameters affecting the performance of rammed earth structures, such as resisting parameters and applied loads, consist randomness. In classical deterministic analysis, which is still widely used for design of different types of structures, uncertainties are not included in structural design. In this approach loads and resistance parameters are considered by their worst cases. To apply probabilistic design in analysis of the structures random variables and limit states shall be defined precisely. Random variables are modeled with appropriate probability distribution functions and distribution parameters to consider the uncertainty of the design parameters. Limit-state functions, on the other hand, are used to define the failure domain [15]. The applicable probability distribution and required parameters for different resistance random variables are summarized in Table 1.

Table 1: Resistance random variables parameters.

Random Variables	Distribution	Mean Value	COV (%)	References
Compressive strength of URE walls (MPa)	Lognormal	1,1.5,2, and 2.5	35	[2, 5, 12, 16]
Compressive strength of CSRE walls (MPa)	Lognormal	1,1.5,2, and 2.5	22	
Humidity effect of External walls	Uniform	0.75	19.24	[20,22]
Humidity effect of Internal walls	Uniform	0.95	3.03	
Smoothness factor of External walls	Normal	0.9	5	-
Smoothness factor of Internal walls	Normal	0.95	2	
Erosion factor of URE walls	Gumbel	0.016	25	[20]
Erosion factor of CSRE walls	Gumbel	0.005	20	

For ideal rammed earth construction, the maximum dry density is achieved by compacting the earthen materials at its optimum moisture content to have a high compressive strength. Most investigations show that there is correlation between the unconfined compressive strength and dry density of samples. The correlation factor between density and compressive strength of both URE and CSRE walls was considered 0.8 based on data extracted from past researches [2, 12].

2.1. Compressive Strength

Compressive strength of materials, govern the resistance RE structures subjected to of gravity loads and noncyclic transient loads. It is almost impossible to estimate the compressive strength of the earthen materials without experimental tests. The recommended values for unconfined compressive strength of RE material is varied between 1 to 2 MPa as per different guidelines [4, 7, 8]. In this study, four different mean values of 1, 1.5, 2 and 2.5 MPa were considered to assess the effect of compressive strength on the performance of rammed earth structures.

3. Load Random Variables

In deterministic approach, the minimum probable capacity of structure shall be greater than the demand of the possible acting loads considering their maximum probability. In contrast, in reliability analysis, the loads are modeled with their probabilistic data and characteristics to limit the failure probability to a desired value.

In this study, dead load as a permanent load and live, snow and wind loads as the transient loads were applied to the rammed earth walls. The height of stories (walls) was considered 3 m and the walls was supposed to be unreinforced. It was assumed that slenderness of walls have been controlled and did not have effect on the design. This type of structure is usually constructed in areas with low seismic risk. Therefore, the earthquake loads were not considered in this study. Random variables contributed to the applied loads are presented in Table 2, with the applicable probability distribution and modeling parameters.

Table 2: Loads random variables parameters.

Random Variables	Distribution	Maximum Value (1)	Mean Value	COV (%)	References
Dead Load (kN/m^2)	Normal [20]	-	1.5	7	[19]
Density (kg/m^3)	Normal [20]	-	1900	7	[2, 5, 12, 14, 16]
Live Load (kN/m^2)	Gumbel [20]	2	-	29	[21, 22]
Roof Live Load (kN/m^2)	Gumbel [20]	1	-	29	[21, 22]
Snow Ce Factor	Lognormal [23]	-	1.2	10	[22]
Snow Cs Factor	Lognormal [23]	-	1.1	10	[22]
Snow Ct Factor	Lognormal [23]	-	1.1	10	[22]
Snow Ground Load (kN/m^2)	Lognormal [23]	3	-	10	[22]
Wind Speed (km/h)	Gumbel [24]	130	-	50	[22]

(1) The maximum values are obtained based on 2 % probability of exceedance in 50 years lifetime of structures

4. Analysis

4.1. Analysis Method

The probabilistic analysis is described by a vector, $\mathbf{x} = [x_1, x_2, ..., x_n]$ to represent the random variables, and the performance of the structure is shown by the limit state function, $g(\mathbf{x})$. The limit state function is defined in terms of resistance (R) and demand (S) of the structures. The failure probability (p_f) based on the defined limit state function can be calculated by computing the following integration [25]:

$$p_f = P[g(\mathbf{x} \le 0)] = P[R \le S] = \int_{g(\mathbf{x} \le 0)} f_x(\mathbf{x}) d\mathbf{x} \tag{1}$$

Where $f_x(\mathbf{x})$ is the joint probability density function of the random variables, **x**, and the integral is calculated over the failure domain $\Omega \equiv \{g(\mathbf{x}) \le 0\}$.

Finding the joint probability density function of random variables is difficult and almost impractical due to difficulty of calculation of joint probability multiple integral. Hence, some approximate methods such as First-Order-Reliability-Method (FORM were proposed and used. In FORM the random variables, x, are transformed into standard normal space, u, and the other steps of calculation is done in this space. The limit state function is presented in this space by $G(\mathbf{u}) \le 0$. The integral in Eq. (1) is calculated over the failure region defined in standard normal space. The design point, $\mathbf{u}^*$ in the standard normal space is the point located on the limit state function $G(\mathbf{u}) \le 0$ with the maximum probability density. In other words, the design point is an approximation to the limit state surface with nearest distance to origin in the failure domain.

The probability approximated at design point is:

$$P[g(\mathbf{x} \le 0)] \approx \Phi(-\beta) \tag{2}$$

Where b is the reliability index and is equivalent to the distance from the origin to the calculated design point and Φ is the standard normal cumulative density function. The limit state in FORM is approximated by linearizing at the design point and can be shown as follows:

$$G(\mathbf{u}) \cong \nabla G(\mathbf{u}^*)(\mathbf{u}-\mathbf{u}^*) = \left\|\nabla G(\mathbf{u}^*)\right\|(\beta - \boldsymbol{\alpha}\,\mathbf{u}) \tag{3}$$

where $\nabla G(\mathbf{u}^*)$ is the gradient vector at the design point and a is the unit negative gradient vector at the design point pointing toward the failure domain and is calculated by:

$$\boldsymbol{\alpha} = -\nabla G(\mathbf{u}^*) / \left\|\nabla G(\mathbf{u}^*)\right\| \tag{4}$$

Iterations are required to calculate the values of $G(\mathbf{u})$ and $\partial G / \partial \mathbf{u}$ at the trial points. The improved Hasofer-Lind-Rackwitz-Fiesler (iHLRF) algorithm [26] was used to select the direction vector and step size.

4.2. Sensitivity Measure

The results obtained from a reliability model are sensitive to the random variables. The importance of random variables and the sensitivities of the reliability analysis to the random variables can be extracted by the results of FORM [27]. The “Gamma Importance Vector”, g, which is applicable for correlated random variables, measures the importance of random variables by using the following equation:

$$\boldsymbol{\gamma} = \frac{\boldsymbol{\alpha}\,\mathbf{J}_{\mathrm{u,x}} D}{\left\|\boldsymbol{\alpha}\,\mathbf{J}_{\mathrm{u,x}} D\right\|} \tag{5}$$

Where D is the diagonal matrix of standard deviation of random variables (x) in their defined space, a is the unit negative gradient vector at the design point and $\mathbf{J}_{\mathrm{u,x}}$ is the Jacobian of the transformation.

4.3. Limit State functions

The limit state functions are used to define the limit between failure and safe domains. In this study, the limit states were defined in terms of resistance and demand of the structures as stipulated by Eq. (1). The demand on the structure, R, due to applicable loads, which were Dead Load (D), Live Load (L), Snow Load (S) and Wind Load (W) were obtained and compared with the resistance of structures which is calculated based on following equation:

$$R = (C_{s,x}\ H_y)[t(1-Sm_y)(1-E_x)] \tag{6}$$

where Cs, H, t, Sm and E denoted the compressive strength of materials, the humidity factor, wall thickness, the smoothness and erosion effect, respectively. The suffix, x, was used for the stabilized and unstabilzied materials, and the suffix, y, was used to distinct between external and internal walls.

Four distinct limit state groups were defined and are shown in Table 3. The load cases considered in each group and applicable position of walls are also presented. The first two groups were used for internal walls only. The reason was that the sustained gravity loads on the external walls considering the loading area of walls are normally smaller than internal walls while their thickness are normally equal to or greater than them. “DLSW” and “DW” are applicable for external walls only. The former limit state used for external walls was a representation for maximum applied load and the later was a representation for overturning.

Table 3: Limit States Groups.

Limit State Group	Load Case				Applicable Wall	
	Dead	Live	Snow	Wind	Internal	External
“DL”	X	X	-	-	X	-
“DLS”	X	X	X	-	X	-
“DLSW”	X	X	X	X	-	X
“DW”	X	-	-	X	-	X

The limit states were named based on following rule: LS-THK-SP-ST-MAT. Where LS denoted the applicable loads and limit state group, in which “D”, “L”, “S”, and “W” stand for dead, live, snow and wind load, respectively. THK represented wall thickness and varied between 125, 150, 200 and 250 mm for internal walls and 250, 300 and 400 mm for external walls. SP specified the span of the wall is 4 or 5 m. ST indicated the number of stories, 1 for single story and 2 for two-story buildings. The height of stories were considered 3 m. Finally, MAT denoted the type of material. “URE” and “SRE” stand for unstabilzied rammed earth material and cement stabilized rammed earth material, respectively. For example the limit state “DLSW-250-4-1-SRE” represented 4 m span external wall of a single story rammed earth building with stabilized materials and 250 mm thickness and subjected to dead, live, snow and wind load.

4.4. Target Reliability and Failure Probability

The objective of reliability analysis is to keep the reliability index larger than target reliability index (β_T) which prohibits the probability exceeds the target limit. Since now, no limit has been proposed for the target reliability of rammed earth structures for reliability analysis. In this study, referring to Chinese unified standard for reliability design of building structures GB 50068 [28], the target reliability of 3.7 was chosen. The selected value was for structures with brittle behavior categorized in second class structures including houses, offices, etc., which are not categorized in important structures (first class) nor temporary ones (third class).

5. Results and Discussion

5.1. Reliability Analysis of Internal Walls

FORM was used to perform reliability analysis of RE walls under different limit states. The reliability index then was compared target reliability index of 3.7 (equivalent to 0.01078 % failure probability). The reliability analysis of the internal walls on the first limit states group (Limit State “DL”), indicates that for most situation the reliability index is greater than the target value. This quite great reliability index shows that the walls could be used safe to support the normal permanent dead load and transient functional live load value. Only for 125 mm thick URE walls with 1 MPa compressive strength, supporting two-story buildings, the design is unsafe.

The results for URE walls supporting two-story buildings for walls subjected to snow loads in addition to the dead and live load (“DLS” limit states) are demonstrated in Fig. 1. It proves that, the reliability index is highly correlated to the compressive strength and wall thickness. By increase in span length and increase in number of stories, the index is decreased as expected. The same results were observed for “DL” limit states. As can be observed, for URE structures with 1 and 1.5 MPa compressive strength, the reliability index does not pass the target limit in most cases. Using thicker walls (200 mm or more) and higher strength material (2 MPa as recommended by Burroughs [9, 10]) improves the behaviour of RE walls.

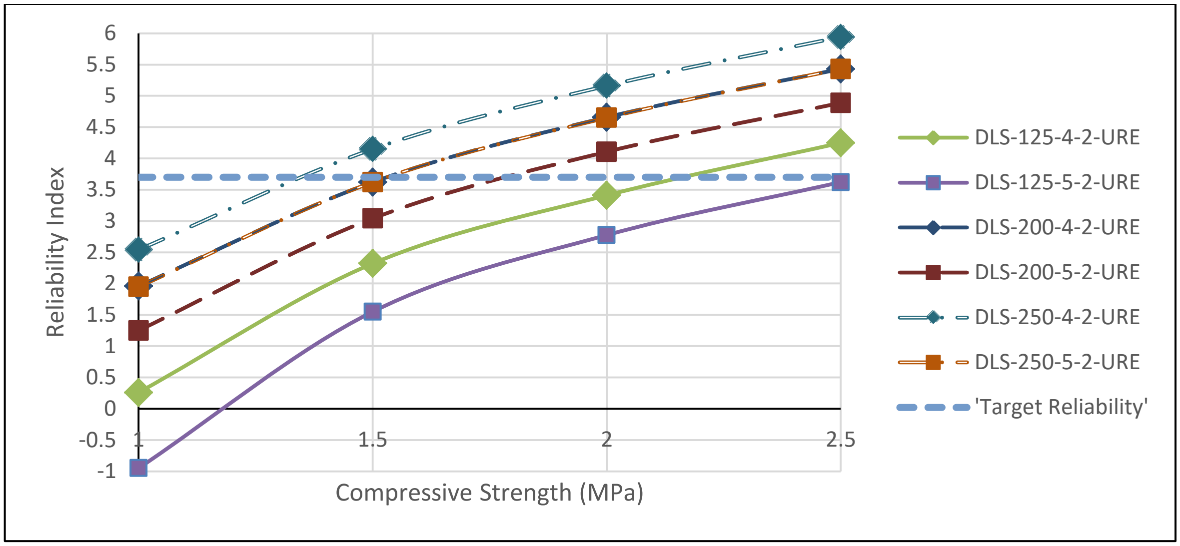

Fig. 1: Reliability index for internal walls (“DLS” limit states, URE structures).

Results of reliability analysis of CSRE structures, show that the reliability index do not pass the target value for the 125 mm thick walls with 1 MPa strength supporting two-story buildings. Considering that usually the compressive strength of the rammed earth structures stabilized with cement are far greater than 1 MPa, this case is not frequently used. It is also concluded that using thick internal walls especially for the case of high strength materials, is followed by very conservative and uneconomical design.

5.2. Reliability Analysis of External Walls

The reliability indices for “DLSW” limit state show that CSRE materials is greater than target limit for all considered situations. This case is also valid for URE materials supporting single-story buildings. The results of reliability analysis of two-story buildings for URE structures can be seen in Fig. 2. The conclusions proved for URE internals walls are also valid for external walls subjected to this limit state group.

The results for analysis of external walls for the limit state “DW” (as shown in Figs. 3 for URE structures) show that, the reliability index is relatively constant for different compressive strength. This is valid for both URE and SRE materials. The reliability index is also close for the same wall thickness regardless of the material type (URE or SRE). It is concluded that the applied gravity load instead of other strength parameter controls the design. The reliability analysis shows 250 mm thick walls for 5 m span do not behave well; however, 300 mm external walls perform safe in this limit state group. The thicker walls (for example 400 mm) are followed by very conservative design.

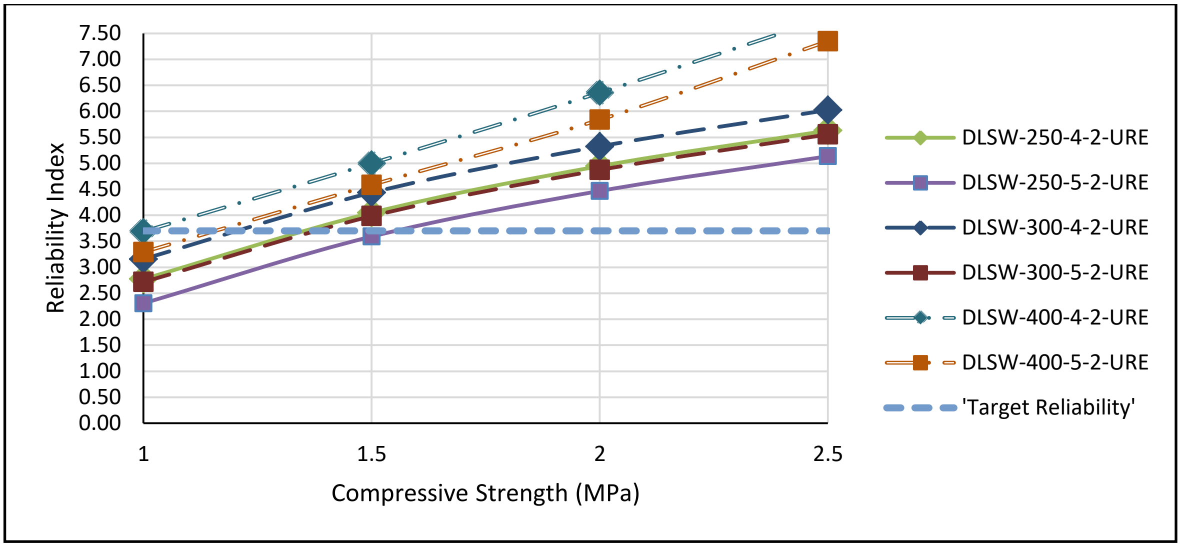

Fig. 2: Reliability index for external walls ("DLSW" limit states, URE structures).

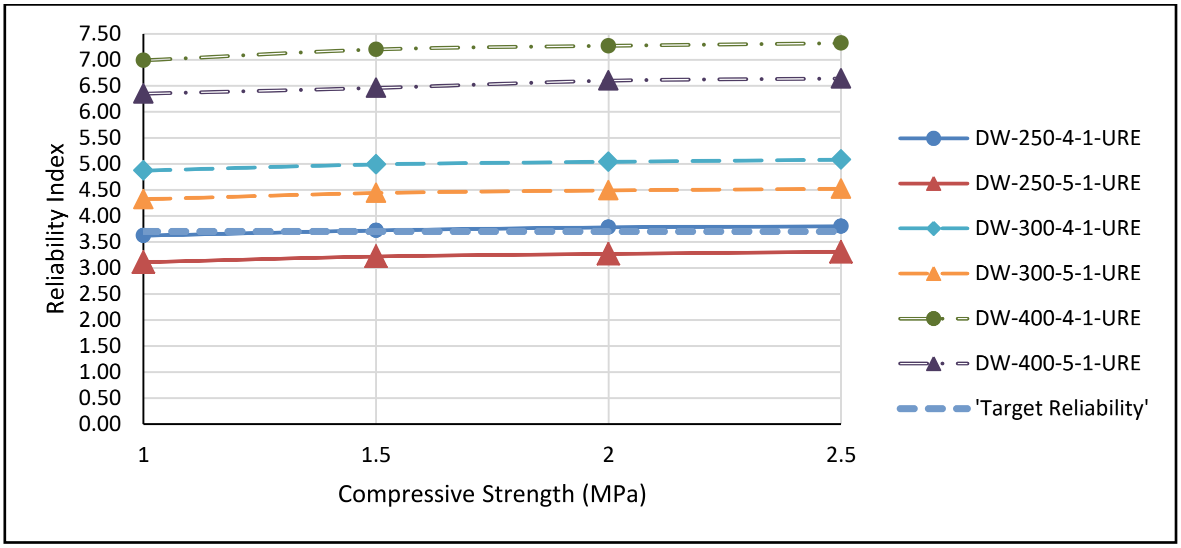

Fig. 3: Reliability index for external walls ("DW" limit states, URE structures).

5.3. Sensitivity Analysis

The importance vectors calculated for internal walls indicated that the compressive strength was, by far more, the most important uncertain parameter. The other important resistant random variables were smoothness effect and humidity. In the loading random variable groups, live and snow load factors were the most effective ones. Fig. 4 shows the obtained Gamma importance vectors for external walls. For walls subjected to circumstance of "DLSW" limit states, compressive strength and humidity factor are the most important random variables. However, for "DW" limit state, it can be observed that the reliability index is most sensitive to wind speed and by a far distance following by density. In the mentioned limit state dead load and density are categorized in resistance parameters, since the sign of their values in importance vector is negative. It was seen that in other limit state groups they were acting as load.

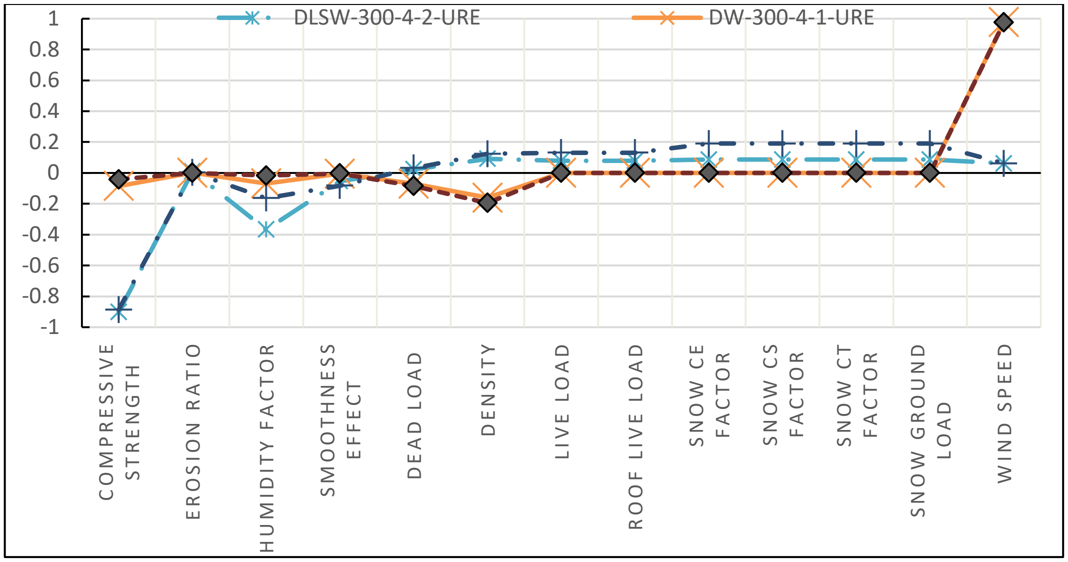

Fig. 4: Gamma importance vector for external walls.

6. Conclusions

In this study, uncertainty was included in the design of rammed earth structures. The reliability index and failure probability of them, then, were calculated and compared with the target reliability index to check the safety of design. The affecting parameters on the strength of the structure and loads were consider with proper probability distributions and correlations. Defined limit states considering the environmental load combinations were used in reliability analysis. The sensitivity analysis was used to define the random variables that have most effect on the failure probability of such structures.

- The RE structures stabilized by cement, thanks to their low erosion probability, less susceptibility to loss of strength due to change in relative humidity have better performance than URE structures under similar conditions.
- The recommended thickness of walls presented in the guidelines are very conservative for moderate loading conditions. On the other hands, walls subjected to heavy snow loads or high speed required thicker walls than recommended by codes.
- The wall thickness and gravity loads on the walls control the reliability analysis of the external walls subjected to high speed wind loads. The compressive strength and type of walls have very low effect in this case.
- Based on sensitivity analysis, compressive strength of materials and wind speed are most important random variables for walls subjected to snow and wind load, respectively.

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