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Frequency Response of a Microcantilever Immersed in Fluid

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Abstract - The micromechanical systems include devices and technology such as actuators and electronic elements on a micrometric scale. A key piece for the development of these systems are the micro cantilevers, which mechanical and dynamic features allow to design sensors and actuators, among others. However, the dynamic response of a microcantilever is altered when it is immersed in a fluid, such as water or even air. Thus, the physical models that describe their behavior in normal conditions (vacuum) do not apply. Considering this condition, this article presents the physical models that describe the behavior of the microcantilevers in fluids (water and air) through the analysis by finite elements. The results show that the density and viscosity of the fluid alter both the oscillation amplitude of the microcantilever and modify the oscillation frequency. Nevertheless, the behavior of the microcantilever in vacuum and air are approximate.

***Keywords*:** MEMS, FEA, Microcantilever, Fluid, Frequency.

1. Introduction

The microelectromechanical systems (MEMS) are microsystems that include micromechanical sensors, actuators and micromechanical circuits in a more complex structure which can carry out control tasks, detection of physical or chemical magnitudes turning them into electrical signals and handling on a micro scale. According to some authors [1], this type of technology does not only considers the miniaturizations of mechanical components or the creation of silicon devices but also the technology used for their manufacturing. This technology is rapidly becoming one of the most promising ones. Among its benefits, are the reduction in size of the components, the lessening in energy consumption and that it allows the development of possible uses in different fields such as automobile industry, telecommunications, biomedical uses, among others. As a basic element for the development of the micromechanical devices, there are the microcantilevers, as they are of great interest for investigators thanks to its long range of possible uses. Microcantilevers are microscale beams made with a solid material like silicon with a fixed end and a free one which let them flex and oscillate. As a potential use, they can come in several shapes as they have just two operation modes through which they can carry out detection and actuation tasks. Microcantilever applications range from medicine [2], energy harvesting [3], to telecommunications (radio frequency microelectromechanical system) [4], [5]. One of the two operation modes of microcantilevers is the dynamic mode. This is given when the microcantilever oscillates while monitoring the frequency shift that results when are added small masses to the microcantilever or when any outside force acts on it. However, the physical models that describe the behavior of microcantilever in the dynamic mode are just applicable in ideal conditions especially when the microcantilever is in the vacuum, not under the effects of fluids (air, water etc.) that surround it. Therefore, a variety of articles try to examine this issue, for instance John Elie Sader's paper [6], where there is a detailed theoretical analysis about the response, expressed in frequency, of a microcantilever immersed in a viscous fluid, presenting analytic and numerical formulas that are useful to designers and users of microcantilevers. This study was carried out using an atomic force microscope. Maali et al. they carried out experimental measurements and compared them with theoretical results concluding that there is a high difference between the experimentally dissipated energy and the analytically calculated energy for the superior oscillations modes [7]. Cornelis A. Van Eysden and John E. Sader proposed a series of analytic formulas to calculate the resonance frequency of a microcantilever immersed in fluids, which viscosity is despicable, that works for any oscillation mode [8]. In this article, the problem to calculate the oscillation frequencies of a microcantilever immersed in two fluids using the finite element analysis (FEA) is considered. Specifically, frequencies in air and water are calculated and then the obtained results are compared with the ones analytically obtained. In the dynamic mode, the microcantilever behaves like a resonator which is just a system capable of oscillate in certain frequencies (Eigen frequencies or natural frequencies) and can reach resonance. The classical Euler–Bernoulli beam theory is used to describe or model the oscillating behavior of beams on micrometric and nanometric scale from where equation 1 is derived, which is the mathematical expression that calculates the oscillating frequency of microcantilever (vacuum) in function of its dimensions and physical characteristics (see Fig. 1) [9].

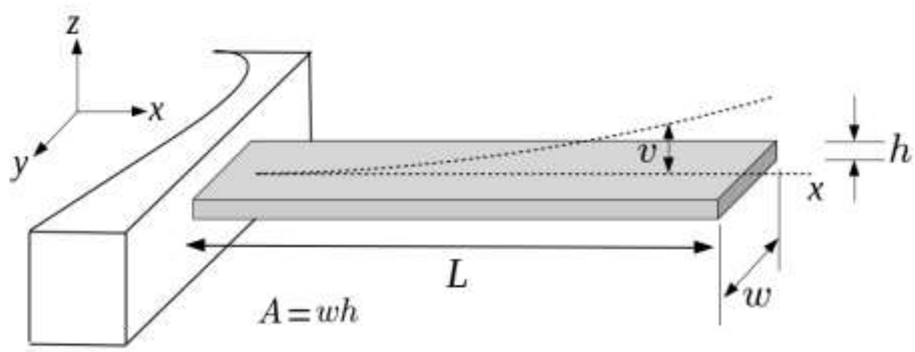

Fig. 1: Scheme of a microcantilever in which the cross section area A, the length L, the thickness h, and the width w, are shown. Source: Authors.

In equation (1), w is the width, h is the thickness, L is the length, ρ is the density and E is Young's modulus of the microcantilever. The sub index n indicates that it corresponds with the umpteenth oscillation mode, therefore λ_n is the umpteenth positive root of the equation $1 + cosh(\lambda_n)cos(\lambda_n) = 0$.

$$f_n = \frac{\omega_n}{2\pi} = \frac{{\lambda_n}^2}{2\pi}\sqrt{\frac{Eh^2}{12\rho L^4}} \tag{1}$$

However, the dynamic response of the microcantilever is seriously affected by the fluid in which it is immersed [11]. To model this phenomenon on microcantilevers, some theoretical models have been set, that consider this issue and have been experimentally valuable [7], [12]. The following described model starts from these assumptions [13]:

The cross section of the microcantilever is uniform along its length and it is rectangular. The length L of the microcantilever is far larger than its width w and this latter is far larger than its thickness. The microcantilever is made of an elastic material and the amplitudes of oscillations are very short comparing them with the dimensions of the microcantilever. The dissipation of the energy produced by the structure is despicable. The biggest energy dissipation is caused by the fluid. The fluid is totally free and it is incomprehensible.

The expression that determines the umpteenth natural frequency of the microcantilever immersed in a fluid is:

$$f_{vn} = f_n\left[1 + \frac{\pi\rho_v\omega}{4\rho h}\Gamma(k)\right]^{-1/2} \tag{2}$$

Where ρ and ρ_v are the densities of the microcantilever and the fluid respectively [8]. In the calculus of the frequency f_{nv}, f_n the frequency in vacuum of the umpteenth oscillation mode takes part. The term $\Gamma(k)$ depends on λ_n of the oscillation mode and the microcantilever geometry.

$$\kappa = \lambda_n \frac{w}{L} \tag{3}$$

$$\Gamma(\kappa) = \frac{1 + 0.74273\kappa + 0.14862\kappa^2}{1 + 0.74273\kappa + 0.35004\kappa^2 + 0.058364\kappa^3} \tag{4}$$

2. Methodology

The simulations were carried out by using the COMSOL Multiphysics software because it allows the finite element analysis FEA and the designing according to different uses at engineering and physics. COMSOL offers a series of modules that can be combined between themselves to do a great variety of analyses. To analyze the microcantilever dynamics the most of the studios were made within the frequency domain. The respective analyses were made within the Euler-Bernoulli beam theory. In all simulation processes, the same dimensions were kept for the microcantilever in study. These were taken from the Angel Cruz's work [14], who suggested a first analysis with a polysilicon microcantilever, the physical characteristics and dimensions are seen in the table 1.

Table 1: Dimensions and physics characteristics of the microcantilever.

Characteristic	Symbol	Value
Density	ρ	2320 Kg/m^3
Young's modulus	E	169 x10^9 Pa
Poisson's ratio	v	0:22
Length	L	200 μm
Width	w	50 μm
Thickness	h	1.5 μm

At the beginning, the first three natural frequencies of the microcantilever were evaluated immersed in two fluids, water and air, which densities were 1000 Kg/m3 and 1.2 Kg/m3 respectively. For this purpose, the geometry of the Fig. 2 was built on the simulator; this geometry consists of a sphere in where a six-side solid is placed. The solid represents the microcantilever while the sphere represents the fluid that surrounded it. The dimensions of the solid correspond with the dimensions of the microcantilever described in the table 1 and the radius of the sphere is 300 μm.

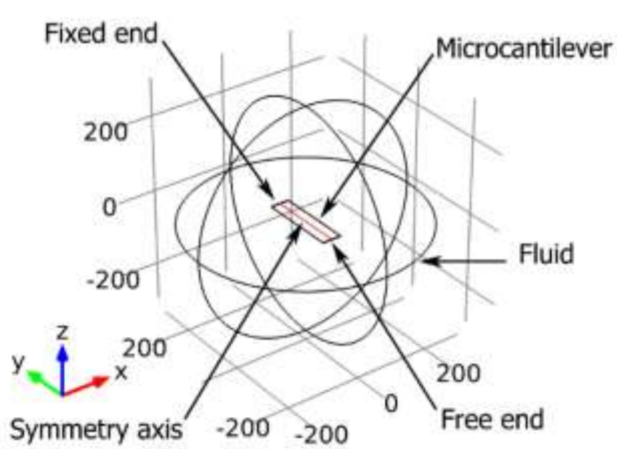

Fig. 2: Geometry used for the simulations of the microcantilever in the presence of a fluid. View of the microcantilever inside the sphere, the symmetry axis, the free end and the fixed end.

Once the calculus of the first three natural frequencies were made, a frequency sweep for the microcantilever was carried out, oscillating in each one of the fluids within the range from 1 Hz to 1 MHz. The oscillation's amplitude of the microcantilever was evaluated by exerting a force 1× 〖10〗 ^ (-6) N on the free end of the microcantilever, taking 500 samples every 2 KHz. The exerted force was parallel with the Z-axis.

3. Results and Discussion

The table 2 contains the oscillation natural frequencies of the microcantilever of the first three modes in the two fluids (water and air).

Table 2: Oscillation natural frequencies of the microcantilever immersed in water and air. f_T, theoretical value; f_S, simulated value; E % percent error.

	Air			Water		
Mode	f_T (kHz)	f_S (kHz)	E %	f_T (kHz)	f_S (kHz)	E %
1	51.362	51.890	1.028	14.993	16.345	9.017
2	322.156	325.130	0.923	99.652	106.100	6.470
3	902.953	913.060	1.119	301.617	315.790	4.699

The Fig. 3 shows the results of the sampled frequency of the microcantilever in vacuum, water, and air. It was noticed that the microcantilever does not only have frequencies in which the oscillation amplitude is maximum (resonance) but it also has frequencies in which the amplitude of oscillation is practically null, they are called antiresonance frequencies. These antiresonance and resonance frequencies are alternating frequencies. It was also noticed that for the thicker fluid, water, the resonance frequencies of the microcantilever are closer, causing that, for the evaluated range of frequencies, the microcantilever in the air only reaches the third oscillation mode. It is worth mentioning that the difference between the simulated results of the microcantilever working in vacuum and air is not too much, such as it is remarked in Fig. 3a. This suggests that is possible to accelerate the simulation process and reduce computing resources by approximating a microcantilever working in air to a microcantilever working in vacuum.

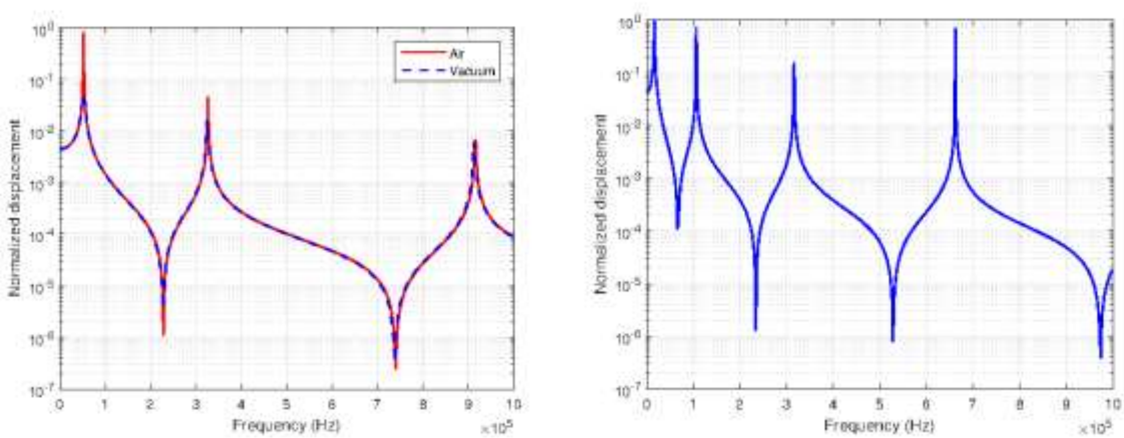

Fig. 3: Frequency sweeps of the microcantilever in different mediums. (a) Frequency sweep in the vacuum and in the air. (b) Frequency sweep in water. Source: Authors.

The simulated results of the microcantilever oscillating in air and water exhibit a percent error. Particularly in water, the percent error increased up to 9%. This difference between simulated and theoretical data can be due to that the simulation does not satisfy the assumptions of the model used for the calculus of the frequencies of the microcantilever immersed in a fluid. The dimension proportions of the microcantilever are not either fulfilled since the length, L, is not far larger than the width, w, but only 4 times larger. Besides, in the simulation the fluid is compressible because the zones that are next to the microcantilever do experiment shifts in pressure (Fig. 4).

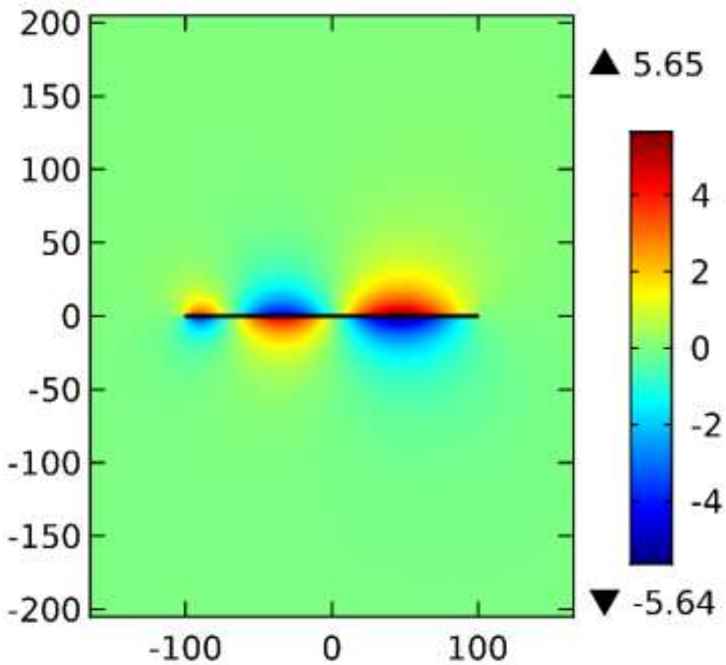

Fig. 4: Effect on pressure of the third natural oscillation mode of the microcantilever on the surrounding fluid (water). The fixed end of the microcantilever is located on the right side of the figure and the scale of this is micrometer. Pressure experimented in the areas close to the microcantilever (Pa). Source: Authors

4. Conclusion

The analysis by finite element is an useful tool to solve very complex problems such as the calculus of oscillation frequencies of a microcantilever immersed in a fluid. In this particular problem, not only the movement of a mechanical structure was analyzed but also other factors such as pressure and viscosity were analyzed. The effect produced by the fluid, when the resonance frequencies of the microcantilevers shift, can be used in determining the density of the fluid. The results given by the simulation and the theoretical model suggest that the increase of the density of the fluid, in which the microcantilever is immersed, raises the percent error between the simulated data and those obtained analytically. As a process of this work, several simulations of microcantilever immersed in different fluids should be realized to determine the relation of oscillation frequency and fluid density and, in this way, prove the possibility to measure the density through microcantilever.

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