

Calculating Rainfall Flow in a Substation Hydraulic System

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Abstract - This paper describes a methodology for estimating rainfall discharge to an oil-water separator. The methodology is part of a comprehensive program for substation maintenance analysis developed by Hydro-Quebec. One purpose of hydraulic systems in substations is to collect rainwater with residual oil through a series of different types of oil collection pits connected to a system of conduits in turn connected to an oil-water separator. The oil collection pits and the separator are designed to protect the environment from oil spills or leaks. The methodology developed determines discharge from the pits through outlets of various design using a method of explicit time integration of analytical and empirical equations. One-dimensional Saint-Venant equations were solved by successive approximation using the under-relaxation method to model flow in the conduit network. A fictitious substation is used as an example and the accuracy of the results is analyzed and confirmed.

Keywords: rainfall flow, oil collection pit, pit outlet, conduit network

1. Introduction

Hydraulic system management during rainfall is a common technical operation when operating a substation. Electrical equipment likely to spill or leak oil is protected by oil collection pits that prevent oil from entering the environment. The different oil collection pits are connected to an oil-water separator by a conduit network.

When it rains, water enters the oil collection pits, mixes with oil residues and ultimately flows through the conduit network into the separator. This oil-water mixture must not overflow through the walls of the oil collection pits, and the separator must have the capacity to separate oil and water at a given discharge. The model described in this paper was developed to test existing substations with respect to these two parameters and to provide help for the design of new substation oil collection pits. The project is part of a comprehensive program analyzing the functioning of substations which was developed by Hydro-Quebec. As experimentation using real substations is highly complex, hydraulic and mathematical models were used to check our results. The program code included verification of the fundamental law of conservation of mass. A series of test were carried out to compare analytical solutions of corresponding equations where possible.

2. Basic concept of the model

Calculations are performed by integration over time, with rain intensity at each time step predetermined. Rainwater entering the oil collection pit creates flow from the pit into a conduit network and from there into the separator. As there is only a small amount of residual oil, it does not affect the flow; hence the asymptote that the liquid can be considered water is acceptable. It should be noted that flow in the conduits does not influence flow in the pits. In other words, water can only flow out of the pit through the pit outlet and not into the pit—as expected since the typical oil collection pit design is a gravity flow system.

3. Flow into the oil collection pits

An oil collection pit is a reservoir of complex geometry with inclined surfaces and internal channels (Fig 1.). The bottom of the pit has a high point and a low point, with a flow outlet at the low point. Our model is designed for oil collection pit geometry analysis only and not for design or optimization purposes. One model assumption is that fluid flows from the high point to the low point with depth the same in all channels, an assumption ensured by correct design of the gravity flow

channel system. Pits can connect to each other (Fig. 1). Most pit channels are filled with gravel. Some channels may have a perforated pipe to facilitate drainage to an outlet.

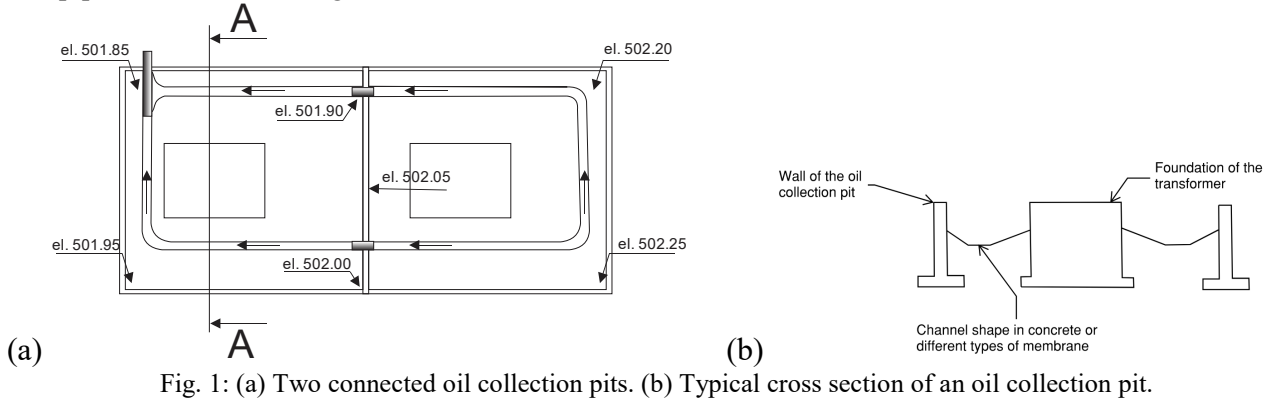


Fig. 1: (a) Two connected oil collection pits. (b) Typical cross section of an oil collection pit.

The volume of water in the pit at time t is:

$$V_b^t = V_b^{tp} + V_{in}^{\Delta t} + V_{rain}^{\Delta t} - V_{over}^{\Delta t} - V_{out}^{\Delta t} \quad \Delta t = t - tp \quad (1)$$

where

- V_b^{tp} : volume of water in the pit in previous time step tp , m^3
- $V_{in}^{\Delta t}$: volume of water flowing into the pit from another pit over time step Δt , m^3
- $V_{rain}^{\Delta t}$: volume of rainwater falling into the pit over time Δt , m^3
- $V_{over}^{\Delta t}$: volume of water overflowing the pit wall, m^3
- $V_{out}^{\Delta t}$: volume of water flowing out of the pit over time Δt through the outlet, m^3

The volume of rainwater falling into the pit over time Δt is:

$$V_{rain}^t = A_b \int_{tp}^t I dt = 0.5(I^{tp} + I^t)A_b \Delta t \quad (2)$$

where

- A_b : pit area, m^2
- I : rain intensity, m/s

The volume of water flowing out of the pit over time Δt through the outlet is:

$$V_{out}^t = \int_{tp}^t Q dt = 0.5(Q^{tp} + Q^t)\Delta t \quad (3)$$

where

- Q : discharge through the pit outlet, m^3/s

Water height in the channels at any moment in time can be determined from the volume of water in the pit at that moment assuming (a) rainwater enters the channels immediately (no flow to electrical devices or other elements) and (b) water height is the same in all channels (uniform flow):

$$\sum_{j=1}^J (A_c L_c)_j = V_b \quad (4)$$

where:

- J : number of channels in the pit
- A_c : wetted cross-sectional area of channel j , m^2
- L_c : length of channel j , m

In Eq. (4), the wetted cross-sectional area, in the case of gravel-filled channels, is divided by the porosity of the gravel. If there is a perforated pipe in the channel, then its effect on the final wetted-section surface is also considered geometrically (Fig. 5c).

If water height in the channels exceeds the height of the pit wall, the pit will overflow. The volume of water overflowing is then:

$$V_{over}^t = V_b^t - \sum_{j=1}^J (A_{c\ max} L_c)_j \quad (5)$$

where:

$A_{c\ max}$: full channel j cross-sectional area, m^2

Rain flow is unlikely to result in overflow of a pit wall but the equation can be used for analyses of other situations where such a possibility exists—extinguishing a fire, for example.

Note that flow during any one time step is not steady state. Flow velocity in the channels can be calculated approximately only, as follows:

(a) for gravel-filled channels, flow velocity (m/s) is calculated according to Darcy's law:

$$v = KS \quad (6)$$

where:

K : hydraulic conductivity of gravel, m/s

S : slope of channel

(b) for channels without gravel, flow velocity (m/s) is calculated according to the Manning formula:

$$v = \frac{R^{\frac{2}{3}} \sqrt{S}}{n} \quad (7)$$

where:

R : hydraulic radius, m

n : Manning's roughness coefficient, $s/m^{1/3}$

For a channel with a perforated pipe along its length, discharge is calculated using Eq. (7) only for that part of the flow that goes into the pipe, since it is only through the pipe that water can enter an outlet. Water height in the perforated pipe is the same as in the pit.

Pit outlets can have different configurations, but the hydraulic flow can generally be described as flow through a weir or an orifice and the general formula for weirs and orifices can be used [1]:

$$Q = \int_{H_1}^{H_2} \sqrt{2gy} w dy \quad (8)$$

where:

H_1 : hydraulic head at the high point of the weir, m

H_2 : hydraulic head at the bottom point of the weir, m

g : free fall acceleration, m/s^2

y : distance from the free surface to the elementary layer of water, m

w : cross-sectional width of the elementary layer, m

dy : height of the elementary layer, m

Application of Eq. (8) to various pit outlet designs is discussed below.

3.1 Flow leaving the pit through a gabion

In this design, flow enters the gabion from channels that are generally rectangular in cross-section (Fig. 2a). For such rectangular channels, the general formula of Eq. (8) is transformed as follows (Fig. 2b):

$$Q = \int_{\frac{v^2}{2g}}^{h + \frac{v^2}{2g}} \sqrt{2gy} w dy = \frac{2}{3} W \sqrt{2g} \left(\left(h + \frac{v^2}{2g} \right)^{\frac{3}{2}} - \left(\frac{v^2}{2g} \right)^{\frac{3}{2}} \right) \quad (9)$$

where:

W : channel cross-sectional width, m

h : height of flow, m

For channels of other cross-sectional shapes or channels ending in a weir, integration over height depending on channel width at different levels is required.

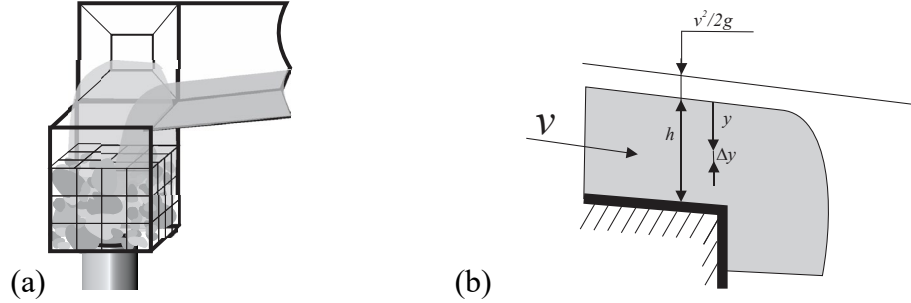


Fig. 2: (a) Pit outlet design with gabion. (b) General formula for weirs and orifices applied to outlet with gabion.

3.2 Outflow through horizontal orifice in the wall

Horizontal orifices in the wall are a common design for pits filled with gravel and connected to other pits (Fig. 3). This type of outlet is considered an orifice in the reservoir if it is completely submerged in water or a weir if the water level is below the top edge of the orifice. The stones are pressed quite tightly against the orifice, reducing the surface available for water drainage depending on gravel porosity. Water level in the downstream pit affects flow in the upstream pit. Used in this case is Dubuat's formula [2], which regards the overflow as composed of two parts: an upper part treated as flow over a weir and a lower part treated as flow through a submerged orifice. The combination of the two parts yields:

$$\begin{aligned}
 Q_u &= \eta \int_{\max(h_1 - D, 0) + \frac{v_1^2}{2g}}^{h_1 - h_2 + \frac{v_1^2}{2g}} \sqrt{2gy} \, w \, dy & h_2 < D \\
 Q_u &= 0 & h_2 \geq D \\
 Q_l &= \eta \int_{h_1 - h_2 + \frac{v_1^2}{2g}}^{h_1 + \frac{v_1^2}{2g}} \sqrt{2g(h_1 - h_2)} \, w \, dy & h_2 < D \\
 Q_l &= \eta \int_{h_1 - D + \frac{v_1^2}{2g}}^{h_1 + \frac{v_1^2}{2g}} \sqrt{2g(h_1 - h_2)} \, w \, dy & h_2 \geq D
 \end{aligned} \tag{10}$$

$$Q = Q_u + Q_l$$

where:

- Q_u : discharge, upper part of flow, m^3/s
- Q_l : discharge, lower part of flow, m^3/s
- η : porosity of gravel in the channel
- h_1 : water height in upper flow pit, m
- h_2 : water height in lower flow pit, m
- v_1 : water velocity in upper channel, m/s
- D : orifice diameter, m

Note that no correction factors are included in Eq. (10) but they can be added if necessary. When water flows from a pit into a hatch, only the upper part of the flow is considered, and thus Eq. (10) is converted to Eq. (9) for a circular weir.

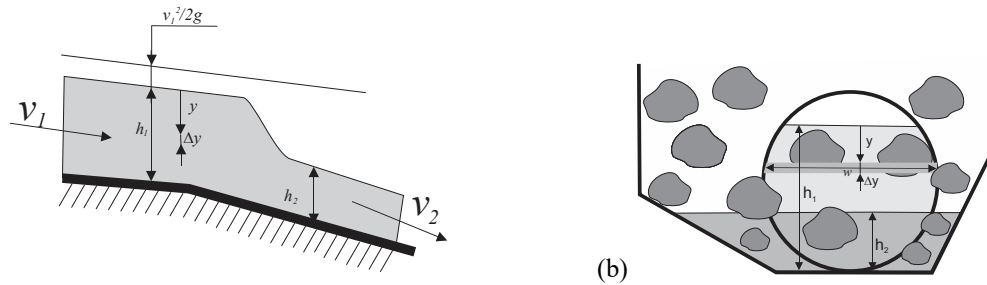


Fig. 3: (a) Horizontal orifice in the wall with connected pits. (b) Dubuat's formula applied to orifice in the wall.

3.3 Outflow through vertical orifice in channel bottom

Discharge through a vertical orifice (Fig. 4) is as follows:

$$Q = \eta \frac{\pi D^2}{4} \sqrt{2gh} \quad (11)$$

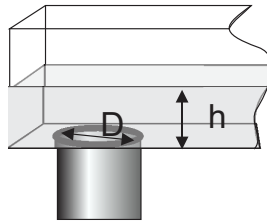


Fig. 4: Vertical orifice in channel bottom.

3.4 Outflow through a perforated pipe

A perforated pipe is often installed along the channel to increase the outflow from a gravel-filled pit. Sometimes a short perforated pipe is installed at the low point of the pit (Fig. 5).

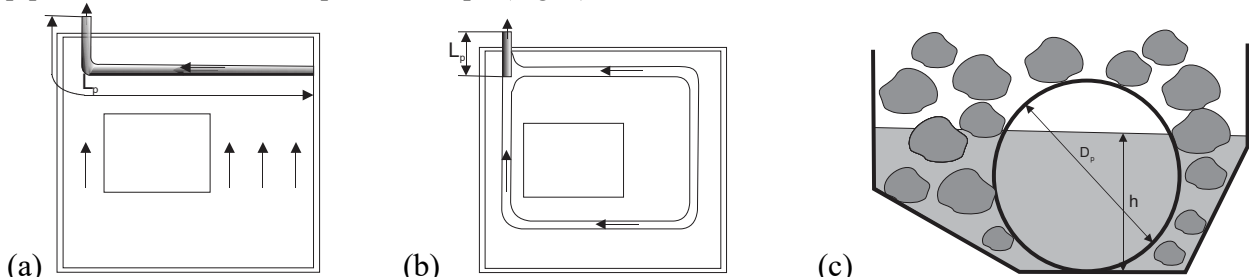


Fig. 5 (a) Pit design with perforated pipe along channel. (b) Pit design with short perforated pipe. (c) Cross section of channel with perforated pipe.

Outlet discharge is calculated with Eq. (9) using the velocity in the perforated pipe, calculated with Eq. (6). Outflow at the pit outlet is limited by inflow to the pipe, which is:

$$Q_{pipe\ max} = \sqrt{2gh} \left(A_{pp} L_p \frac{P_{pw}}{\pi D_p} + A_{pw} \eta \right) \quad (12)$$

where:

A_{pp} : total open area per unit length of pipe, m^2/m

L_p : length of perforated pipe, m

D_p : diameter of perforated pipe, m

P_{pw} : wetted perimeter of perforated pipe, m
 A_{pw} : wetted area of perforated pipe, m²

The last term in Eq. (12) is important only for short perforated pipes, where discharge through the main working surface of the pipe is comparable to discharge at the free edge.

4. Flow in conduit network

Hydraulic calculations for the conduit network are performed using the same methodology as in the SWMM5 software [3]. One-dimensional Saint-Venant equations are used to describe unsteady flow:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \quad (13)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial \left(\frac{Q^2}{A} \right)}{\partial x} + gA \frac{\partial H}{\partial x} + gAS_f + gAS_l = 0 \quad (14)$$

where

A : wetted cross-sectional area of the conduit, m²
 Q : discharge, m³/s
 t : time, s
 x : distance along the conduit, m
 H : hydraulic head, m
 S_f : friction slope
 S_l : local energy loss per unit length of conduit

These equations are supplemented with initial and boundary conditions. The initial condition is a dry conduit before rainfall. The boundary conditions are: (a) for hatches connected to the pits, inflow discharge is as specified in section 3 above; (b) for the outlet to the separator, a free fall flow condition is used. Eqs. (13)-(14) are solved using a method of successive approximations with under relaxation [3].

5. Example: analysis of a fictitious substation

For purposes of this presentation, we use a fictitious substation with only two oil collection pits as an example to demonstrate necessary data and results obtained. The peculiarity of this case is that the oil collection pits have the same hydraulic characteristics, differing only in the design of the outlets. Fig. (6) shows the substation configuration and tables (1)-(2) the geometric characteristics. Fig. (7) shows rainfall intensity.

Table 1: Pit parameters

Pit 1						Pit 2					
Surface, m ²		32				Surface, m ²		32			
Outlet type		Gabion				Outlet type		Horizontal orifice Ø 0.2 m			
Channel	Length, m	Weight, m	Slope, %	Manning's coefficient, s/m ^{1/3}	Wall height, m	Channel	Length, m	Weight, m	Slope, %	Manning's coefficient, s/m ^{1/3}	Wall height, m
1	3.32	1.41	1.04	0.013	0.8	1	3.32	1.41	1.04	0.013	0.8
2	4.68	0.76	1.27	0.013	0.8	2	4.68	0.76	1.27	0.013	0.8
3	3.32	0.76	1.27	0.013	0.8	3	3.32	0.76	1.27	0.013	0.8
4	4.68	2.01	0.92	0.013	0.8	4	5.68	2.01	0.92	0.013	0.8
5	1.06	2.01	2.00	0.013	0.8						

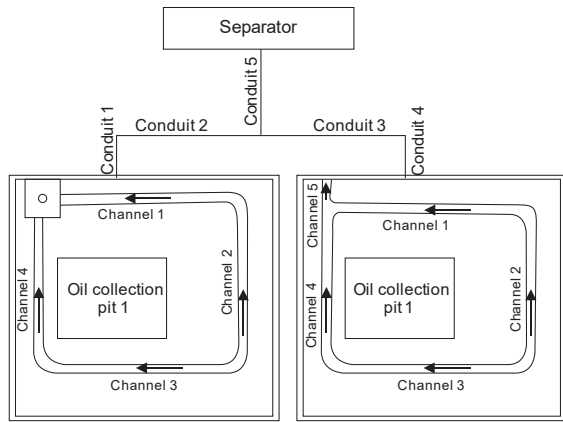


Fig. 6 Example of possible substation configuration.

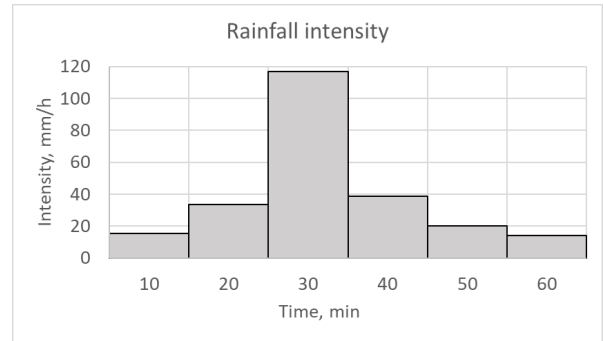


Fig. 7 Rainfall intensity.

Table 2: Parameters of conduit network

Conduit	Length, m	Diameter, m	Upstream elevation, m	Downstream elevation, m	Slope, %	Manning's coefficient, s/m ^{1/3}	Loss coefficient
1	3	0.2	97.19	97.15	1.33	0.013	0
2	10	0.3	97.00	96.95	0.5	0.013	0
3	14	0.3	97.94	97.87	0.5	0.013	0
4	3	0.2	98.00	97.94	2.00	0.013	0
5	21	0.3	96.90	96.78	0.57	0.013	0

The simulation results clearly show the importance of accurate calculations for outlets. As Fig. (8) shows, water flows from a pit with a gabion much faster than from a pit with an orifice in the wall, increasing the flow rate in the conduit system and into the separator.

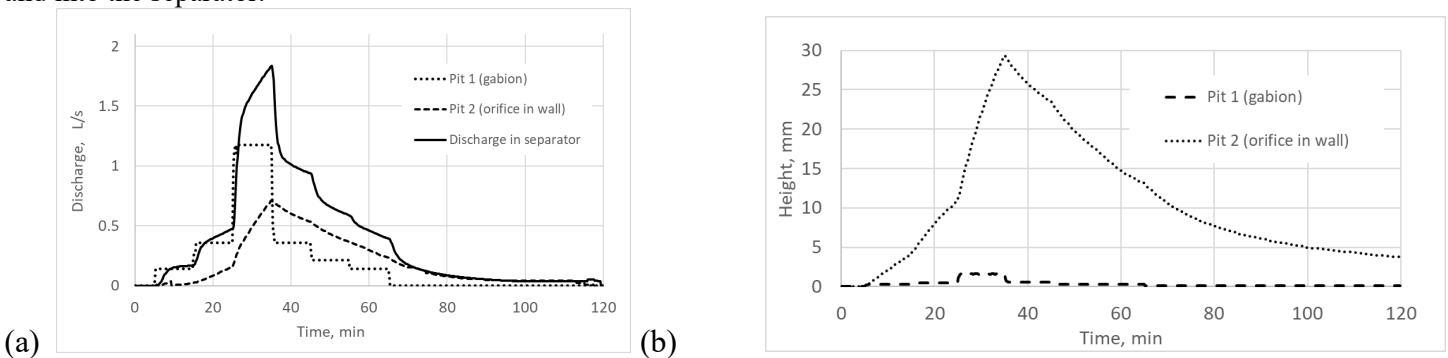


Fig. 8: Flow parameters: (a) water discharge at pit outlets and separator inlet; (b) water height in pit channels.

During the first 30 minutes (Fig. 8a), discharge to the separator is less than flow from pit 1. This is due to the influence of the conduit network. Some substations have long, complex conduit systems that significantly affect flow into the separator. Fulfillment of the law of conservation of mass in the form of volume balance is shown in Fig. 9. Total input volume to each pit during rainfall is the sum of the volumes at the time steps calculated by Eq. 2. Likewise, the sum of the output volumes calculated by Eq. 3 is the total output volume from the pit. Volume flow in the separator at each time step is equal to:

$$V_{sep}^t = \int_{tp}^t Q_{sep} dt = 0.5(Q_{sep}^{tp} + Q_{sep}^t)\Delta t \quad (15)$$

where

Q_{sep} : discharge in separator, m³ (discharge in conduit 5)

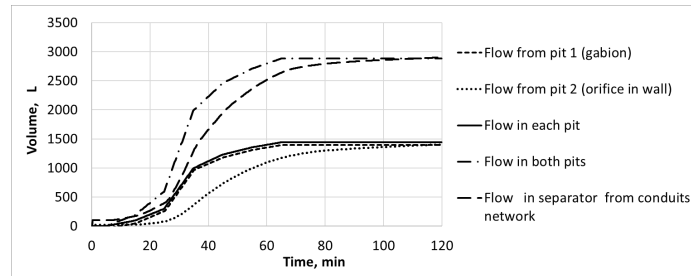


Fig. 9 Balance of volumes.

Fig. 10 shows another type of result checking. A stable discharge every 10 minutes from pit 1 (Fig. 8a) makes it possible to use the Manning formula (Eq. 7) for conduits 1 and 2 (Fig. 6). In this case, the Manning formula is an asymptote for Eqs. (13)-(14) once flow stabilizes. Comparison results are shown in Fig. 10b. Discharges obtained with Eq. 7 were calculated for water heights obtained with Eq. 13 and compared with discharges obtained with Eq. 14. As expected, there were differences only when the nature of the flow changed.

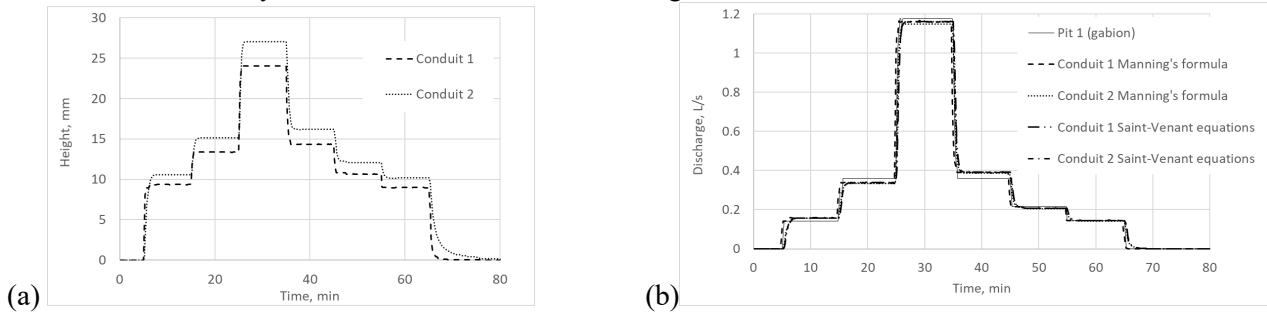


Fig.10. (a) Water height in conduits. (b) Comparison of discharges in conduits calculated by different methods.

4. Conclusion

The methodology developed calculates rainwater discharged into a substation's oil-water separator. Particular attention is paid to accurate calculation of flow through outlets of different design. As shown in the example in the paper, outlet design has a major impact on final discharge from oil collection pits. The empirical and analytical equations used are well known and time-tested, providing a guarantee of results; this is important given that experimentation on real equipment is practically impossible. A volume balance check and comparison with the asymptotic solution using Saint-Venant equations confirmed the reliability of the results. The method is not limited in terms of number and cross-sectional shape of the oil collection pit channels or outlet design. The use of full one-dimensional Saint-Venant equations makes it possible to calculate conduit networks of any complexity: networks with a series of pits connected both directly and via conduits; networks with parallel sections of conduits of different diameters or made of different materials; conduits with negative and zero slopes; wells connected to three or more conduits at different levels; etc. The model is a useful tool for quick and accurate prediction of water discharge to an oil-water separator. The next step will be to model oil-water interaction in the substation hydraulic system, making it possible to simulate liquid with a high oil content and predict the concentration of oil in the flow into the separator.

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