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The Application of Visibility Graph and Graph Attention Network for Urban Smart Grid Short-Term Load Forecasting

Bo Peng[1],
[1]University of Miami
1251 Memorial Drive, McArthur Engineering Building, Coral Gables, FL, USA
bxp547@miami.edu

***Abstract* -** This work proposes a novel approach for short-term load forecasting in urban smart grids using Visibility Graph Attention Network (V-GAT). The V-GAT model effectively addresses the complexities of STLF tasks, including non-linearity and the influence of diverse factors. V-GAT outperforms established Machine Learning and Deep Learning models in terms of accuracy due to its capability to capture intricate data relationships and prioritize informative features. Additionally, V-GAT offers advantages in interpretability and computational efficiency. The superior forecasting accuracy and efficiency make V-GAT a valuable tool for smart grid operators to optimize power management and grid stability.

***Keywords*:** Visibility Graphs, Graph Attention Networks, short-term load forecasting, network statistical measures, Smart Grid

1. Introduction

The traditional power grid is becoming increasingly obsolete, due to growing concerns about technical, economic, and environmental issues. The next generation of electric power system is known as the "Smart Grid (SG)". The global smart grid market was reached at almost 50 billion U.S. dollars in 2022 and is expected to grow at a Compound-Annual Growth Rate (CAGR) of 17.4 percent until 2028, to hit roughly 130 billion U.S. dollars [1]. Compared to traditional grid, the smart grid is expected to allow two-way flows for both electrical power and real-time information and complement the current grid system by including renewable energy resources, such as wind, solar, and hydropower [2]. By leveraging the Internet of Things (IOT) techniques to collect data, the urban smart grid connects a variety of distributed energy resource assets to the power grid and enables effective power management and distribution.

Nevertheless, due to the complex and dynamic nature of the urban electricity system, there are still many challenges ahead in transforming to the smart grid. Among those, perhaps the greatest challenge is predicting the power grid total load (power demand) more accurately and efficiently. In this case, the accuracy of load-forecasting models will have a significant impact on many decisions such as planning schedules of utilities, economic scheduling of generating capacity, system security assessment, and planning for energy transactions [3]. At the same time, the unpredictability character of renewable energy and the urgent need to integrate it into the grid further intensifies the demand for precise power production and load forecasting.

The power grid load forecasting problem is a challenging task due to its complex, non-linear, and non-stationary characters. Hippert et al. [4] have classified load forecast to be short-term (for days), medium-term (for months), and long-term (for years). Short-term load forecasting (STLF) is currently considered the most crucial in day-to-day operations, unit commitment and scheduling, evaluation of grid interchange, and system security analysis. However, high volatility in the load curves makes Day-ahead load forecasting in SGs relatively more challenging when compared to longer duration load forecast [3].

Existing time series forecasting methods encompass a range of approaches, broadly categorized into stochastic distribution-based models, Markov chain-based models, Machine Learning (ML)-based models, and Artificial Neural Network-based Deep Learning (DL) models. Stochastic and Markov chain-based models often exhibit lower prediction accuracy due to their simplifications and assumptions about data dynamics [5], [6]. It's essential to recognize that conventional ML and DL methods, while effective and capable of achieving improved prediction accuracy, have inherent limitations. They may struggle with capturing complex temporal dependencies or handling non-linear relationships in the data [7]. Additionally, their interpretability may be limited, hindering insights into model decision-making processes [8].

In addressing the challenges of STLF in urban smart grids, this work introduces a novel approach that leverages the Visibility Graph (VG) and Graph Attention Networks (GATs). The VG method, recognized for its efficacy in transforming nonlinear time series data into graph structures, allows for the intricate dynamics of electricity load data to be analysed in a new light. This transformation is instrumental in unveiling the underlying patterns within complex dynamical systems, thereby providing a foundation for more accurate forecasting models. Studies such as Lacasa et al. have demonstrated the VG's capability in revealing hidden patterns in nonlinear time series, marking a significant advancement in time series analysis [9]. The application of VG in various domains, including neuroscience, finance, and meteorology, underscores its versatility and effectiveness in handling complex data structures [10] - [12].

Graph Neural Networks (GNNs), and specifically Graph Attention Networks (GATs), have emerged as a breakthrough in processing graph-structured data, leveraging attention mechanisms to dynamically prioritize the most relevant parts of the graph for specific tasks. However, despite their advantages in model performance, including interpretability and efficiency, the application of GATs to VG for regression tasks, particularly in the context of STLF, has been scarcely explored. Our work not only addresses this research gap but also demonstrates the effectiveness of this methodology through a case study on multivariate load forecasting data from Panama, showcasing superior performance over traditional ML and DL models.

This paper's contribution lies in its unique methodology framework that combines the Visibility Graph and Graph Attention Network. By transforming the urban smart grid's load forecasting challenge into a graph-based problem, we propose a more accurate and faster model. This approach not only addresses the limitations of traditional and current nonlinear forecasting models, such as prediction accuracy and computational demands, but also introduces a scalable and interpretable solution. The comparative analysis conducted against established ML and DL models like XGBoost, Random Forest, LightGBM, Bi-LSTM, and GRU further validates the superiority of our proposed methodology, setting a new benchmark for STLF in urban smart grids.

2. Research Framework and Methodology

The proposed research framework is meticulously designed to address the intrinsic complexities of urban smart grid load forecasting. Given the non-linear and non-stationary nature of load demands, coupled with the multifaceted influences of weather conditions and temporal factors, traditional linear models prove inadequate. Consequently, we introduce a novel Visibility Graph Attention Network (V-GAT) model framework (see Fig. 1) that encapsulates the dynamism of the urban electrical load through a visibility graph approach, integrated with Graph Attention Networks (GATs) for superior predictive analytics.

The process starts with data collection and preprocessing. This stage serves as the bedrock of our methodology, where multivariate time series data is meticulously collected. Variables such as temperature, pressure, humidity, and wind characteristics, along with time-related features, are harvested, forming a rich dataset that reflects the complex interplay between time, weather conditions and load demands. Preprocessing, including normalization and missing data handling, is performed with the intent to homogenize the data scale and ensure the robustness of the model against data imperfections.

The application of a sliding window algorithm is a strategic choice, enabling the model to capture temporal dependencies and variations in load patterns over different intervals. This temporal segmentation is pivotal, allowing for an analysis that is both granular and comprehensive, recognizing the transient yet significant fluctuations inherent in short-term load forecasting. The VG construction serves as a transformative step, translating each window of multivariate time series into a graph network where each node represents a unique timestamp enriched with a constellation of features and the target load value. This representation is not merely a data structure but a reflection of the temporal and feature-based relationships that govern the load dynamics, facilitating a nuanced and holistic analysis.

Capitalizing on the latest advancements in GNNs, our framework employs Graph Attention Networks (GATs) to augment the power of VGs. This integration allows the model to dynamically prioritize and learn from the most influential features and temporal relationships, thereby enhancing prediction accuracy. The GATs' attention mechanisms are instrumental in discerning the subtle yet critical patterns that traditional models may overlook [13]. The training and validation phase is conducted with a rigorous and iterative approach, ensuring the GAT model not only learns the underlying patterns of the training dataset but also generalizes effectively to new graph data. Our testing methodology employs a stringent evaluation protocol, utilizing several performance metrics to provide a comprehensive assessment of the model's predictive accuracy. This analysis is critical in benchmarking the model against existing forecasting methods. Beyond quantitative evaluation, our methodology emphasizes interpretive analysis, where we scrutinize the fluctuation of network statistical measures provided by the VGs to better understand the underlying dynamics of the studied systems.

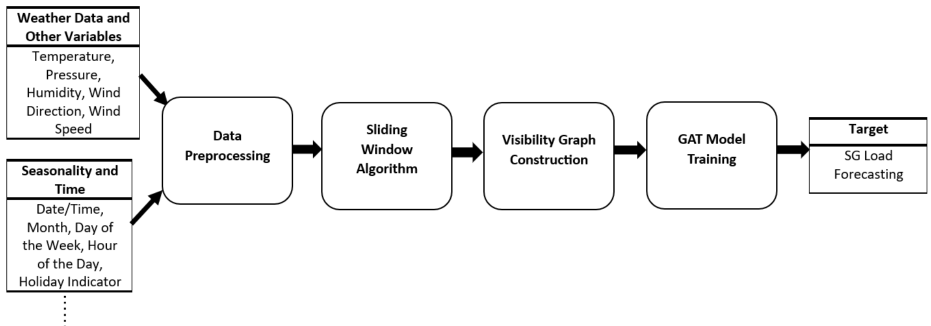

Fig. 1: Proposed Model Framework Flowchart

2.1. Visibility Graph for time series

The Visibility Graph (VG) is a novel analytical tool that translates a time series into a graph, enabling the application of graph theory to investigate the properties of time series data. The mathematical foundation of VG lies in its capacity to map a time series $x(t_i)$, into a network of nodes, where each node $v(t_i)$ corresponds to the data point $x(t_i)$ in the time series. Two nodes $v(t_i)$ and $v(t_j)$ are connected by an edge if, and only if, at least one straight line can be drawn in the bar chart that connects $x(t_i)$ and $x(t_j)$ and does not intersect any intermediate data points. Mathematically, this can be expressed as a visibility criterion given by [9]:

$$x(t_k) < x(t_j) + [x(t_i) - x(t_j)]\frac{t_j - t_k}{t_j - t_i} \qquad i < k < j \tag{1}$$

This criterion ensures that each point in the time series is 'visible' from any other point not obstructed by intermediate data points, thus forming a connection in the graph. As shown in Fig. 2, the Visibility Graph (VG) methodology is applied to a time series consisting of 13 data points, each represented as a bar (Fig. 2 a). The bars are connected by lines representing the visibility from one data point to another. These Lines of Visibility (LoV) ensure that for any two points $x(t_i)$ and $x(t_j)$ a direct line of sight exists if no intermediate data points $x(t_k)$, where i<k<j, obstruct the view. In other words, the line connecting $x(t_i)$ and $x(t_j)$ must not intersect any bars between them.

The resultant undirected VG network (Fig. 2 b) translates these relationships into a graph structure where nodes represent the time points and edges represent the visibility between them. In this network, the nodes are laid out in a manner that preserves the temporal sequence, with the edges illustrating the direct visibility between nodes. The VG thus provides an innovative means of analyzing the structure within a time series by leveraging the principles of graph theory. This approach facilitates the understanding of the intrinsic properties of the data, such as its periodicity and trends, and can also enhance the detection of hidden patterns that are not readily apparent in the raw time series itself [14].

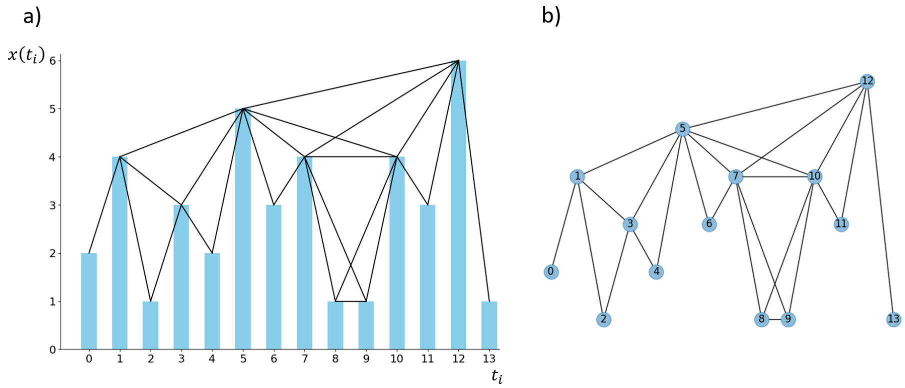

Fig. 2: An example of Visibility Graph a). a random 13 data-points time series in the form of bar chart and corresponding Lines of Visibility (LoV, black lines) of each data-point. b). Associated undirected VG

The selection of VG as the network tool for our study on multivariate load prediction is strategic. The inherent complexity of load time series, influenced by various factors, exhibits nonlinearity and non-stationarity. VG stands out as it does not presuppose linearity or stationarity, enabling it to capture the complex interdependencies and dynamic changes within the time series data [15]. Moreover, the application of VG in our study extends beyond mere representation; it serves as a precursor to employing advanced machine learning techniques, such as GAT. The graph topological structure provided by VG facilitates the use of GAT, which can exploit the intricate relationships between time points to enhance the accuracy of load forecasting.

2.2. Graph Attention Network

Graph Attention Networks (GATs) are a class of neural network architectures that operate on graph-structured data. At the core of GATs is the attention mechanism, which assigns varying levels of significance to nodes within the graph, allowing for the nuanced aggregation of features from neighbors. Introduced by [13], GATs address the challenge of learning from data that is not naturally represented in a Euclidean space but rather in the form of a graph, such as social networks, molecular structures, and in our case, visibility graphs derived from time series data.

The attention mechanism in GATs is computationally analogous to the intuitive process of focusing on the most informative parts of data. For each node, the GAT layer computes attention coefficients that reflect the importance of neighboring nodes' features. These coefficients are learned during training and are used to weigh the neighbors' features accordingly before summing them up. This operation results in new feature representations that are not only a function of neighboring nodes but are also contextually adjusted based on the overall graph structure [16].

In the context of visibility graphs converted from multivariate time series, GATs provide a powerful tool for capturing the relational dependencies that are inherently temporal and contextual. Each node in the visibility graph encompasses the features of the time series at a given timestamp, and the attention mechanism allows the GAT to prioritize which historical data points (nodes) are most relevant for predicting future load values [17]. This is particularly beneficial for load forecasting, where past conditions and patterns can have varying degrees of influence on future load values. By incorporating GATs into our visibility graph-based framework, we enhance the model's ability to learn complex, non-linear dependencies characteristic of load time series, ultimately improving forecasting accuracy and reliability.

3. Short-term Load Forecasting Case Study

2.1. Data Source for the Case Study

The case study we chose, focused on Panama's power system, meticulously compiles a detailed dataset spanning from January 2015 to June 2020. This dataset, comprising 48,048 data points, provides a comprehensive view of the power system's demand and environmental influences [18]. The load forecasting challenge in Panama is emblematic of the broader issues facing power systems globally [19]. The country's electricity demand has been on an upward trajectory, propelled by demographic growth and economic development [20]. The data collected provide an invaluable resource for examining these trends, as they encompass not just the raw electricity demand but also contextual factors like holidays, school cycles, and weather conditions (see Table 1), all of which influence the load on the power grid.

Table 1: Description of original variables in the dataset

Category	Variable	Description	Data Type (Unit)
Time Features	datetime	Date and time recorded in every hour	Time Stamp
	Holiday_ID	Different National Holidays	Categorical
	holiday	Holiday indicator	Binary
	school	School day indicator	Binary
Weather Features	$T2M^c$	Air temperature at 2 meters	Numerical (°C)
	$QV2M^c$	Relative humidity at 2 meters	Numerical (%)
	TQL^c	Liquid precipitation	Numerical (L/m^2)
	$W2M^c$	Wind Speed at 2 meters	Numerical (m/s)
Target	nat_demand	National electricity load	Numerical (MWh)

^{c} Sub-index stands for city, meaning that these variables are available for David, Santiago, and Panama City.

2.2. Data Pre-processing

The raw data underwent meticulous preprocessing to prepare it for the model training. To capture the inherent seasonality and daily consumption patterns within the dataset, we extracted new time features from the existing 'datetime' variable. These features included 'Year', 'Month', 'day of the month', 'day of the week', 'hour of the day', and 'weekend indicator'. Beyond that, we also incorporated several new load features (see Table 2). The existing weather features (T2M, QV2M, TQL, W2M) represent various weather metrics like temperature, humidity, precipitation, and wind speed. However, these features are measured on different scales. To ensure all features contribute equally during model training, a normalization step was applied. This process transformed each weather feature into a value between 0 and 1, eliminating the influence of the original measurement scales.

The model aims to predict the electricity load 24 hours ahead of the current time step (t+24 hours). Therefore, the target variable represents the forecasted load value at this future time horizon. We ensured the target variable was converted to a floating-point data type for compatibility with the machine learning model. There are no missing values on the datasets. Only a few low values on the load were detected due to hourly blackouts and damage in the power grid, but all records were kept.

Table 2: Description of training variables of the forecasting models

Variable	**Future load Forecasts (Target)**	**Current and Previous Load Values**	**Average Load Values**	**Weather Features**	**Time Features**
Description	Predicted load value 24h ahead of current time t	Load values at current time t, 24 h before current time t, and 1 week before current time t	Average of load values measured during the last 24h	Weather related values at 22, 23 and 24h ahead of current time t	Year, Month, day of the month, day of the week, hour of the day, weekend indicator, datetime, Holiday_ID, holiday, school (24h ahead of current time t)

2.2. Model Training and Testing

The efficacy of various forecasting models for short-term load prediction is evaluated. We compare the performance of the proposed V-GAT model against established ML and DL architectures. This study employs three advanced ML models

XGBoost, Random Forest and LightGBM, which are known for their prowess in regression tasks. Most of the important model parameters are fine-tuned in our study.

Two DL models, Bi-directional Long Short-Term Memory (Bi-LSTM) and Gated Recurrent Unit (GRU), which are proficient at learning temporal dependencies within sequential data are included. To provide valuable context for predicting future load, a 24-time-step lagged load feature is incorporated as input for the Bi-LSTM and GRU models.

The V-GAT model leverages a sliding window approach with a window size of 24. This segments the data into consecutive 24-hour chunks. Each chunk is then transformed into a visibility graph (VG), capturing the relationships between data points within that window. The VG representation is subsequently fed into a GAT model for training. The GAT is adept at prioritizing informative features and relationships within the VGs, ultimately enhancing the model's capacity to predict future load values. The data that fit into our models undergoes a chronological split for training (70%) and testing (30%) purposes. This ensures the model is trained on historical data and evaluated on unseen data from the future, mimicking real-world forecasting scenarios.

4. Results and Discussion

Two primary metrics were used to evaluate the performance of the short-term load forecasting models: Root Mean Squared Error (RMSE) and Mean Squared Error (MSE). Lower RMSE and MSE values indicate better forecasting accuracy, signifying a model's capacity to predict future load values that are closer to the actual observed values. The dataset also includes an official 'Pre-dispatch Forecast' value, which is also used here for comparison. Additionally, the computational time required by each model for processing the testing data was recorded.

As can be observed in Table 3, the V-GAT model achieved the superior performance across both evaluation metrics. The V-GAT model produced the lowest RMSE value and the lowest average MSE value compared to other forecasting models. This indicates that the V-GAT model consistently produced forecasts that were closer to the actual load values, on average, compared to the other models. This can be attributed to the V-GAT model's ability to capture the complex interdependencies within the data through visibility graphs and selectively learn from the most influential features and relationships via the GAT mechanism. Moreover, the V-GAT model not only leads in forecasting accuracy but also demonstrates significant computational efficiency, boasting the second-fastest testing time among the models evaluated. Such efficiency, paired with its predictive precision, underscores the V-GAT model's potential as a robust and agile solution for short-term load forecasting within SG systems.

Table 3: Results of Models for short-term load forecasting

Metrics / Models	Pre-dispatch Forecast	Xgboost	Random Forest	LightGBM	Bi-LSTM	GRU	V-GAT
RMSE	80.38	53.43	58.73	53.86	33.65	38.94	**29.52**
MSE	6460.77	2854.75	3448.9	2901.31	1132.21	1516.06	**871.71**
Testing Time (s)	-	0.56	2.28	0.38	1.08	0.64	**0.51**

These two plots in Fig. 3 compare the actual load (blue line) to the predicted load (orange line) generated by the V-GAT model for a sample 24-hour period on a weekday (a) and weekend (b). The plots reveal a high degree of correspondence between the predicted and actual load values across both weekdays and weekends, indicating the V-GAT model's effectiveness in capturing the underlying trends and temporal patterns of electricity load.

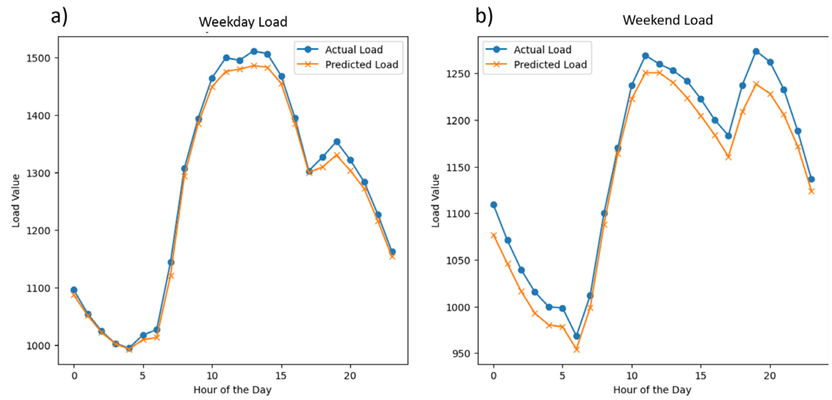

Fig. 3: Actual Load versus V-GAT model forecasting for weekday a) and weekend b)

Fig. 4.a delineates the load dynamics, illustrating the temporal flow of electrical demand with marked peaks and troughs. In parallel, Fig. 4.b encapsulates the clustering coefficient, a pivotal graph-theoretic metric derived from the visibility graph—a transformative representation of the multivariate load time series data with a 24-hour moving window. The clustering coefficient, a measure of the degree to which nodes in a graph tend to cluster together, indicates a probability of nodes to be interconnected [21]. Interestingly, the temporal evolution of the clustering coefficient exhibits a dance with the load values, at times mirroring the load's fluctuations. This dynamic suggests a deeper, nonlinear interplay between the load demand and the VG network's structure. Such graph network statistical measures provide a multifaceted perspective of the load system's behavior, unveiling patterns and dependencies that might be obscured in traditional analyses. Thus, they enrich the proposed model's interpretability, yielding a more nuanced understanding of the underlying system dynamics and bolstering the forecasting model's predictive prowess.

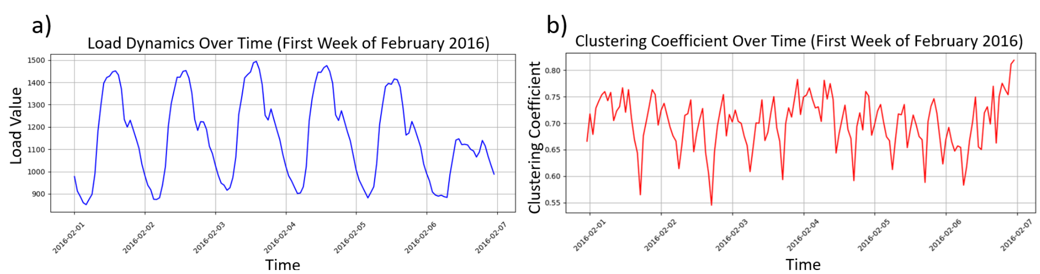

Fig. 4: Comparative Dynamics of Load and Network Structure during the First Week of February 2016. (a) Load Dynamics Over Time. (b) Clustering Coefficient Over Time.

5. Conclusion

This work presented a novel approach for STLF in urban smart grids, leveraging the transformative power of VGs and GATs. The proposed V-GAT model effectively addresses the inherent complexities of load forecasting tasks, including non-linearity, non-stationarity, and the influence of diverse factors like weather and temporal patterns.

The V-GAT framework outperformed established ML and DL models like XGBoost, Random Forest, LightGBM, Bi-LSTM, and GRU in terms of prediction accuracy. This superior performance is attributed to V-GAT's capability to capture intricate relationships within the data through VGs and prioritize informative features and dependencies using the GAT mechanism. Furthermore, V-GAT offers advantages in computational efficiency and interpretability compared to traditional DL models. The model's efficiency in processing STLF data makes it suitable for real-time applications within smart grids.

In conclusion, the V-GAT model presents a valuable tool for smart grid operators. Its superior forecasting accuracy and short computational time making it a valuable tool for grid operators to optimize power generation, distribution, and maintain grid system stability, ultimately contributing to a more reliable and efficient SG infrastructure. Future research can explore incorporating additional features or optimizing the V-GAT architecture for even better performance.

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