

Real-Time Algebraic Estimation of Variable Torque and Parameters in Controlled Synchronous Motors

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Abstract - This paper introduces a novel parameter and mechanical disturbance estimator for a permanent magnet synchronous motor. The estimator, based on the algebraic estimation method, accounts for the temporal variation of most parameters and the non-constancy of load torque. The primary objective is to integrate the estimated parameters into a controller to track trajectories for regulating the direct axis current and motor angular velocity. This integration enhances the robustness of the controller against parametric variations and unmodeled mechanical dynamics. Computational simulations are conducted to validate the performance of both the controller and the designed estimator.

Keywords: Synchronous Motors, Energy Conversion Systems. System Identification, Torque Estimation, Taylor Polynomials.

1. Introduction

Due to climate change, there is a trend towards replacing conventional transportation powered by internal combustion engines with electric motors. Electric motors offer benefits over internal combustion engines, such as reduced emissions of polluting gases, higher efficiency, and lower maintenance costs [1]. Among the most commonly used electric motors for this purpose is the Permanent Magnet Synchronous Motor (PMSM), replacing the induction motor in this field [2]. Several studies have applied the PMSM as the driving source for electric vehicles (EVs), as shown in [3], where a fuzzy controller with sliding modes is proposed to regulate the speed of the PMSM. In [4], a braking torque limit is suggested to improve the regenerative braking of the EV. [5] proposes replacing the mechanical differential, commonly used in various types of vehicles, with a differential based on the use of PMSM. Adopting this motor for EV is not a new topic but remains highly relevant for transportation improvements.

Ideally, all dynamic systems possess constant parameters that do not change during their operation, simplifying both their analysis and control. However, when this ideal scenario does not materialize, complications arise. One way to address this situation is by using estimation techniques to approximate system parameters [6]. In the case of PMSMs, there are various studies where parameter estimation is addressed. For example, in [7], the Recursive Least Squares technique is employed to identify parameters and faults of the PMSM coupled to a transmission. Another estimation technique for PMSMs is the Affine Projection Algorithm, as demonstrated in [8], where this technique is simulated and implemented. Additionally, the Extended Kalman Filter has been proposed as an effective parameter estimation technique, as outlined in [9], where parameter estimation of the motor including the estimation of a constant load torque is performed.

An important challenge faced by some parameter estimation techniques is the difficulty in guaranteeing accurate and real-time estimation. As an alternative to the techniques mentioned above, the Algebraic Parameter Estimation stands out, as presented in [10]. This technique has been applied to other types of electric motors, as demonstrated in [11], where parameters and variable load torque are estimated in a DC Shunt motor. In the case of the PMSM, Algebraic Estimation was used in [12], where it was experimentally performed in one of the examples included. Another case in which this method is applied is in [13], where the estimation of the electric and mechanical parameters of the motor is detailed, including the

constant load torque, in addition to implementing a method restart to detect parameter variations. In [14], only the estimation of variable load torque using Algebraic Estimation is presented.

This article introduces a design for parameter and variable load torque estimation in a PMSM using Algebraic Estimation. It assumes most parameters vary, except for the number of pole pairs. Additionally, the load torque is approximated as a polynomial function over small time intervals. The design includes closed-loop estimation by developing a dual-component controller for tracking trajectories in the direct axis current and motor angular velocity.

2. Parameters estimation and variable torque

The following equations represent the dynamic model of a PMSM in the d-q reference frame:

$$L_d \frac{di_d}{dt} + R_s i_d - L_q n_p i_q \omega = u_d \quad (1)$$

$$L_q \frac{di_q}{dt} + R_s i_q + L_d n_p i_d \omega + n_p \lambda_m \omega = u_q \quad (2)$$

$$J \frac{d\omega}{dt} + b\omega + \tau_L = \frac{3n_p}{2} [\lambda_m i_q + (L_d - L_q) i_d i_q] \quad (3)$$

where L_d and L_q are the direct and quadrature axis inductances, respectively, R_s is the stator resistance, n_p is the number of pole pairs, λ_m is the magnetic flux of the permanent magnet, J and b are the moment of inertia and viscous damping factor respectively, and τ_L is the load torque applied to the motor. The motor state variables are i_d and i_q , representing the direct and quadrature axis currents respectively, and ω , which is the motor angular velocity. The control inputs are u_d for the direct axis and u_q for the quadrature axis [15].

2.1. Estimation of electrical parameters

Firstly, considering the electrical subsystem of the PMSM with unknown parameters Eqs. (1)–(2). It is established that the only parameter that cannot vary over time and is known in advance is the number of pole pairs n_p . To eliminate dependence on the system's initial conditions, both expressions are multiplied by $t - t_i$, where t is the independent time variable and t_i is the initial estimation time. The resulting expressions are integrated with respect to time.

$$L_d \left[\Delta i_d - \int_{t_i}^t i_d dt \right] + R_s \int_{t_i}^t \Delta i_d dt - L_q n_p \int_{t_i}^t \Delta i_q \omega dt = \int_{t_i}^t \Delta u_d dt \quad (4)$$

$$L_q \left[\Delta i_q - \int_{t_i}^t i_q dt \right] + R_s \int_{t_i}^t \Delta i_q dt + L_d n_p \int_{t_i}^t \Delta i_d \omega dt + n_p \lambda_m \int_{t_i}^t \Delta \omega dt = \int_{t_i}^t \Delta u_q dt \quad (5)$$

Inspecting the previous expressions, it is seen that there are more unknowns than equations. To solve this problem, Eq. (4) is integrated with respect to time twice to obtain the appropriate number of equations to solve a linear system. In the case of Eq. (5), the integration with respect to time is done three more times. With this action, the following linear equation systems are obtained

$$A_d \theta_d = B_d, \quad (6)$$

where $\theta_d = [L_d, R_s, L_q n_p]^T$ and the elements of the matrices A_d and B_d are

$$\begin{aligned}
a_{11,d} &= \Delta i_d - \int_{t_i}^t i_d dt, & a_{12,d} &= \int_{t_i}^t \Delta i_d dt, & a_{13,d} &= \int_{t_i}^t \Delta i_q \omega dt, \\
b_{1,d} &= \int_{t_i}^t \Delta u_d dt, & a_{(i,j),d} &= \int_{t_i}^t a_{(i-1,j),d} dt, & b_{i,d} &= \int_{t_i}^t b_{i-1,d} dt.
\end{aligned}$$

with $i = 2, 3$ and $j = 1, 2, 3$. In the case of the quadrature axis equation, its system of equations is

$$A_q \theta_q = B_q \quad (7)$$

where $\theta_q = [L_q, R_s, L_d n_p, n_p \lambda_m]^T$ and the elements of the matrices A_q and B_q are

$$\begin{aligned}
a_{11,q} &= \Delta i_q - \int_{t_i}^t i_q dt, & a_{12,q} &= \int_{t_i}^t \Delta i_q dt, & a_{13,q} &= \int_{t_i}^t \Delta i_d \omega dt, & a_{14,q} &= \int_{t_i}^t \Delta \omega dt, \\
b_{1,q} &= \int_{t_i}^t \Delta u_q dt, & a_{(i,j),q} &= \int_{t_i}^t a_{(i-1,j),q} dt, & b_{i,q} &= \int_{t_i}^t b_{i-1,q} dt.
\end{aligned}$$

with $i = 2, 3, 4$ and $j = 1, 2, 3, 4$. Cramer's rule is proposed to solve Eqs. (6)–(7), leading to the following expressions.

$$\hat{L}_d = \frac{\int_{t_i}^t e^{-\gamma \Delta} |\Lambda_{d,1}| dt}{\int_{t_i}^t e^{-\gamma \Delta} |\Lambda_d| dt}, \quad \hat{R}_s = \frac{\int_{t_i}^t e^{-\gamma \Delta} |\Lambda_{d,2}| dt}{\int_{t_i}^t e^{-\gamma \Delta} |\Lambda_d| dt}, \quad \hat{L}_q = \frac{\int_{t_i}^t e^{-\gamma \Delta} |\Lambda_{q,1}| dt}{\int_{t_i}^t e^{-\gamma \Delta} |\Lambda_q| dt}, \quad \hat{\lambda}_m = \frac{\int_{t_i}^t e^{-\gamma \Delta} |\Lambda_{q,4}| dt}{\int_{t_i}^t e^{-\gamma \Delta} |\Lambda_q| dt}, \quad (8)$$

Where $\Lambda_k = \det(A_k)$, $k = d, q$, and $\Lambda_{k,m}$ are the modified matrices to apply Cramer's rule. The absolute value of the determinants is integrated to avoid potential singularities when the function Λ_k crosses zero. On the other hand, multiplying the absolute value of the determinants by $e^{-\gamma(t-t_i)}$ serves as a filter and helps the result converge quicker as long as γ is positive.

2.1. Estimation of mechanical parameters and load torque

For the mechanical subsystem, it must first be considered that the variable load torque can be locally approximated in a small time interval as an n th-order Taylor polynomial of the form

$$\tau_L \approx \sum_{k=0}^n p_k (t - t_i)^k. \quad (9)$$

Replacing Eq. (9) into Eq. (3), along with considering that $L_q = L_d$, and following the same procedure used for Algebraic Estimation presented in the electrical subsystem, we obtain

$$J \left[\Delta \omega - \int_{t_i}^t \omega dt \right] + b \int_{t_i}^t \Delta \omega dt + \sum_{k=0}^n p_k \int_{t_i}^t (t - t_i)^{k+1} dt = \frac{3n_p \lambda_m}{2} \int_{t_i}^t \Delta i_q dt. \quad (10)$$

The size of the equation system required for estimation depends on the order of the Taylor polynomial used. Hence, a general form of this system is:

$$\begin{bmatrix} a_{11,m} & a_{12,m} & a_{13,m} & \cdots & a_{1,n+3,m} \\ a_{21,m} & a_{22,m} & a_{23,m} & \cdots & a_{2,n+3,m} \\ a_{31,m} & a_{32,m} & a_{33,m} & \cdots & a_{3,n+3,m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n+3,1,m} & a_{n+3,2,m} & a_{n+3,3,m} & \cdots & a_{n+3,n+3,m} \end{bmatrix} \begin{bmatrix} J \\ b \\ p_0 \\ \vdots \\ p_n \end{bmatrix} = \frac{3n_p \hat{\lambda}_m}{2} \begin{bmatrix} b_{1,m} \\ b_{2,m} \\ b_{3,m} \\ \vdots \\ b_{n+3,m} \end{bmatrix}. \quad (11)$$

The mechanical parameters and the coefficients of the Taylor polynomial are calculated using the following expressions:

$$\begin{aligned} \hat{j} &= \frac{3n_p \hat{\lambda}_m \int_{t_i}^t e^{-\gamma \Delta} |\Lambda_{m,1}| dt}{2 \int_{t_i}^t e^{-\gamma \Delta} |\Lambda_m| dt}, & \hat{b} &= \frac{3n_p \hat{\lambda}_m \int_{t_i}^t e^{-\gamma \Delta} |\Lambda_{m,2}| dt}{2 \int_{t_i}^t e^{-\gamma \Delta} |\Lambda_m| dt}, \\ \hat{p}_k &= \frac{3n_p \hat{\lambda}_m \operatorname{sgn}(\Lambda_{m,k+3}) \int_{t_i}^t e^{-\gamma \Delta} |\Lambda_{m,1}| dt}{2 \operatorname{sgn}(\Lambda_m) \int_{t_i}^t e^{-\gamma \Delta} |\Lambda_m| dt}, & \operatorname{sgn}(\beta) &= \begin{cases} 1, & \beta \geq 0 \\ -1, & \beta < 0 \end{cases} \end{aligned} \quad (12)$$

The function sgn determines the sign change of the coefficients p_k .

3. Controller design for PMSM

To test the closed-loop estimation, a Proportional-Integral (PI) controller, is proposed to regulate the direct axis current i_d . The controller development starts with the first expression of the dynamic model, Eq. (1). An auxiliary variable v_d , is introduced to facilitate the development and is defined as:

$$v_d = \frac{di_d}{dt} = -\frac{R_s}{L_d} i_d + \frac{L_q n_p}{L_d} i_q \omega + \frac{1}{L_d} u_d \quad (13)$$

The PI controller is proposed as:

$$v_d = -k_{p,d} e_d - k_{i,d} \int_0^t e_d dt, \quad e_d = i_d - i_d^* \quad (14)$$

Here, $i_d^* = 0.1 \sin(2t)$ is the current trajectory for the direct axis, and e_d is the tracking error. To simplify the controller's second part, i_d^* is proposed to be close to zero and non-constant to facilitate the system's parameter estimation while disregarding it later. Now, differentiating Eq. (14), we get:

$$\frac{d^2 e_d}{dt^2} + k_{p,d} \frac{de_d}{dt} + k_{i,d} e_d = 0 \quad (15)$$

To tune the controller gains, a second-order reference system for the closed-loop error dynamics e_d is chosen as:

$$\frac{d^2 e_d}{dt^2} + 2\omega_n \zeta \frac{de_d}{dt} + \omega_n^2 e_d = 0 \quad (16)$$

By comparing Eqs. (15) and (16), the controller gains are determined, ensuring stability when $\omega_n, \zeta > 0$. Thus, the control input for the direct axis is:

$$u_d = L_d v_d + R_s i_d - L_q n_p i_q \omega \quad (17)$$

Continuing with developing the angular speed controller ω , a Proportional-Integral-Derivative (PID) controller is proposed. Starting from Eq. (3), differentiating with respect to time yields:

$$\frac{d^2 \omega}{dt^2} = \frac{3n_p \lambda_m}{2J} \frac{di_q}{dt} - \frac{b}{J} \frac{d\omega}{dt}. \quad (18)$$

Replacing the derivative values, we get:

$$\frac{d^2 \omega}{dt^2} = \frac{3n_p \lambda_m}{2J} \left[-\frac{R_s}{L_q} i_q - \frac{n_p \lambda_m}{L_q} \omega + \frac{1}{L_d} u_q \right] - \frac{b}{J} \left[\frac{3n_p \lambda_m}{2J} i_q - \frac{b}{J} \omega - \frac{1}{J} \tau_L \right] \quad (19)$$

Again, an auxiliary variable is used for the controller's development, defined as $v_q = d^2 \omega / dt^2$. The PID controller is proposed as:

$$v_q = \frac{d^2 e_\omega}{dt^2} - k_{d,q} \frac{de_\omega}{dt} - k_{p,q} e_\omega - k_{i,q} \int_0^t e_\omega dt, \quad e_\omega = \omega - \omega^*, \quad (20)$$

where ω^* is the reference trajectory of the angular velocity for the PMSM, and e_ω is the tracking error of this trajectory. Continuing with the development, differentiating Eq. (20), we have:

$$\frac{d^3 e_\omega}{dt^3} + k_{d,q} \frac{d^2 e_\omega}{dt^2} + k_{p,q} \frac{de_\omega}{dt} + k_{i,q} e_\omega = 0 \quad (21)$$

To select the controller gains, a stable polynomial of the same order as the highest derivative of Eq. (21) is proposed as $P(s) = (s + p)^3$, so that $k_{i,q} = p^3$, $k_{p,q} = 3p^2$ and $k_{d,q} = 3p$. Finally, u_q is solved from Eq. (19). The parameters for both controllers are $\zeta = 2$, $\omega_n = 45$ rad/s, and $p = 100$. The reference trajectory of angular velocity ω^* is defined by:

$$\omega^* = \begin{cases} \omega_1, & 0 \leq t < t_1 \\ \omega_2 + (\omega_2 - \omega_1)\varphi, & t_1 \leq t < t_2 \\ \omega_2, & t > t_2 \end{cases} \quad \varphi = \sum_{i=1}^3 r_i \left(\frac{t - t_1}{t_2 - t_1} \right)^{2+i} \quad (22)$$

where $\omega_1 = 0$, $\omega_2 = 150$ rad/s, $t_1 = 0$, $t_2 = 10$ s, $r_1 = 10$, $r_2 = -15$ and $r_3 = 6$. The following expression represents the proposed load torque for the initial test cases.

$$\tau_L = \begin{cases} 0.125t^2, & 0 \leq t < 2 \text{ s} \\ 0.5 + \sin(t - 2), & t \geq 2 \text{ s} \end{cases}$$

4. Computational simulation results

The motor used for the computational simulation of the parameter estimator and the controller is the Estun EMJ-04APB22, whose values were obtained from [16]. The motor parameters and the proposed variations are shown in Table 1.

Table 1: PMSM parameters used in the computational simulation.

Parameter	Initial value	Variation	Parameter	Initial value	Variation
L_d, L_q	8.5 mH	2% at $t = 0.5$ s; 5% at $t = 3.5$ s; 1% at $t = 7$ s 0% at $t = 9$ s	J	31.69×10^{-6} kg m	2% at $t = 0.7$ s; 3% at $t = 2.5$ s; 1% at $t = 4.5$ s; 0% at $t = 8.5$ s
R_s	2.7 Ω	5% at $t = 1.5$ s; 3% at $t = 4$ s; 1% at $t = 6.5$ s 0% at $t = 8$ s	b	52.79×10^{-6} Nms	2% at $t = 1.5$ s; 4% at $t = 3.5$ s; 0% at $t = 7.5$ s
λ_m	0.0615 Wb	2% at $t = 1$ s; 5% at $t = 3$ s; 1% at $t = 4.5$ s 0% at $t = 7.5$ s			

The simulation results are depicted in Fig. 1, showing the control inputs. Small spikes are observed due to abrupt parameter changes, but their amplitude does not pose a considerable risk.

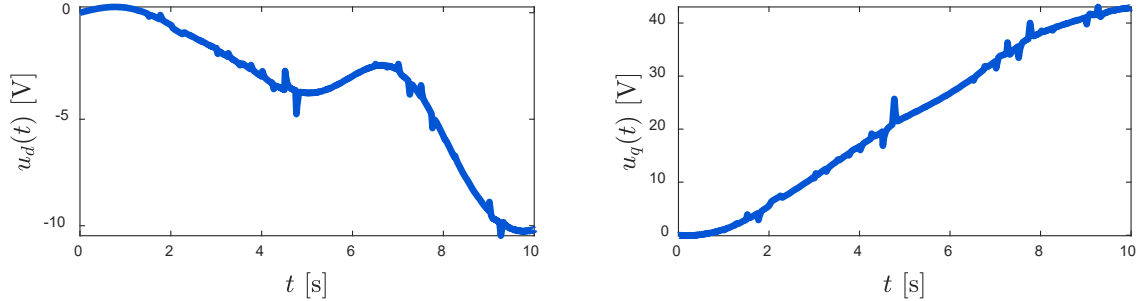


Fig. 1: PMSM control inputs.

Fig. 2 displays the PMSM output signals, which also exhibit spikes. The direct axis current i_d follows the set trajectory, except a spikes couple. Regarding the speed tracking, spikes are visible, but an acceptable trajectory is maintained. Therefore, the controller and the estimator provide acceptable trajectory tracking.

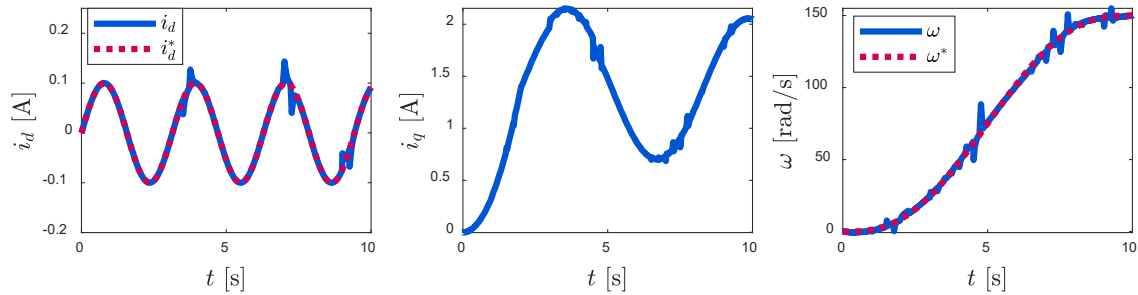


Fig. 2: PMSM closed-loop response signals.

Electrical parameters estimation is depicted in Fig. 3, illustrating a good tracking of the proposed changes in Table 1. A slight phase lag between the actual parameter change and the estimated one is noticeable in all cases. This discrepancy arises because there is a small time interval for estimation, during which the results are updated.

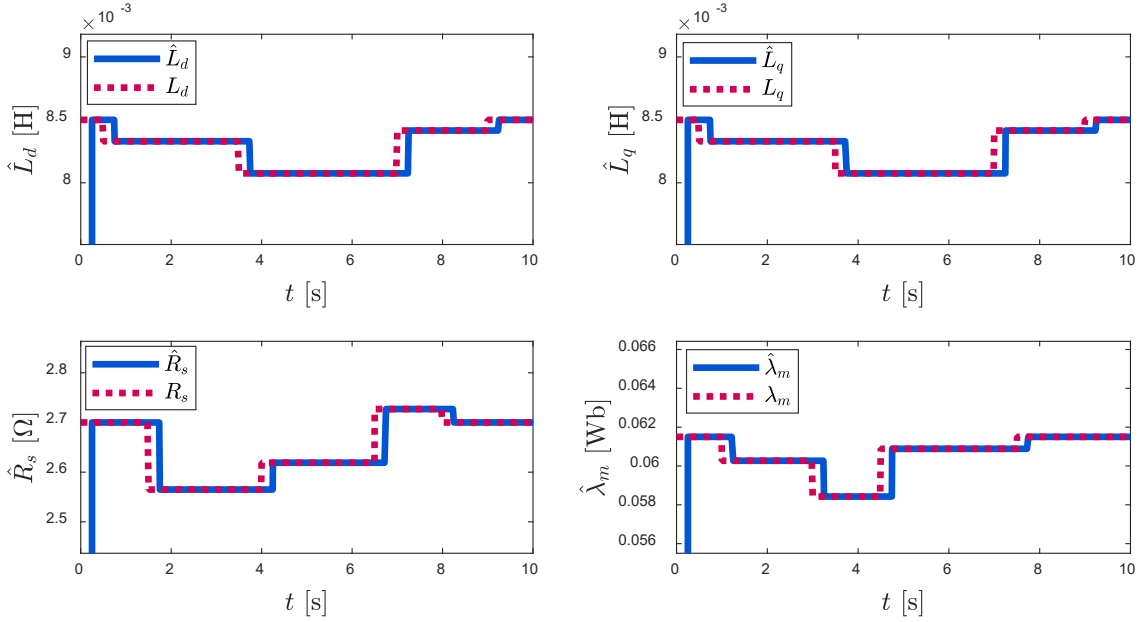


Fig. 3: Estimation of electrical parameters of the PMSM.

Finally, the estimation of the mechanical parameters, as well as the load torque, are depicted in Fig. 4. It can be observed that the estimation of the inertia moment accurately tracks the variations, with only minor deviations. Some errors are noticeable regarding the viscous damping factor, which, although slightly more pronounced, remains acceptable. As for the load torque, estimated using a third-order Taylor polynomial, there is virtually no discernible difference between the proposed and estimated signals.

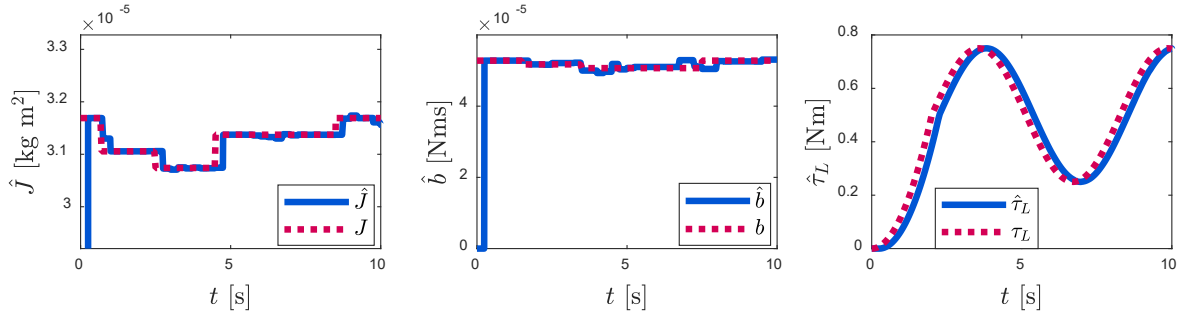


Fig. 4: Estimation of the mechanical parameters of the PMSM and the load torque.

5. Conclusion

In this work, a variable load parameter and touch estimator applied to a permanent magnet synchronous motor was developed using Algebraic Estimation. The estimated parameters and the approximated torque were taken as feedback to a controller that regulates the direct axis current and the angular speed of the motor. The results show good tracking in both the trajectories proposed for the controller and the parameter estimates. A clear area of opportunity is to integrate the estimated parameters more smoothly so there are peaks in the motor responses. Another path that could be followed is to incorporate a machine's or vehicle's dynamics to observe the differences between the estimate and what was obtained in this work.

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