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Model Studies for Dispersion of Effluent Discharges on Eroding Seabed

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Abstract **-** Using an analytical solution of a two-dimensional advection-diffusion equation with a point source on a simple model of seabed depth profile change, the effect of erosion on a sloping seabed upon mixing and dispersion of the outfall effluent discharges in coastal waters is investigated. For near-shore discharges, the maximum value of concentration at the shore is formulated and used as an environmental impact measure that should not be exceeded anywhere along the shoreline. It is found that, the bed erosion increases this maximum value, and installing a multiport diffuser at the end of the outfall long pipeline can supress this increase.

Keywords**:** Advection-diffusion equation, far field model, multiport diffuser, near-shore, sea outfall, shoreline concentration, sloping seabed

1. Introduction

It is generally accepted that the environmental effects of waste effluent discharges from a sea outfall could be kept to a minimum [1]. This can be accomplished by the pretreatment of the waste streams followed by discharge through an effectively designed outfall system which terminates in multiport diffusers [2,3]. For near-shore discharges, due to relatively shallow water depth, the deflected effluent plumes are observed to be heading and spreading towards the shoreline, and may cause a build-up of concentration to higher levels in the coastal waters. Not only are the coastal areas used as fish catchments and conservation areas, some areas are also becoming important developing areas of industry and population. Sandy beaches are popular holiday resorts used by tourists for recreation and swimming.

Physically, the coastal area is a dynamic region where land and sea meet. Generally, it is a sloping sandy beach, but in some places, it is a mountainous coast with rocky sea cliffs, where the waters get deep very rapidly. However, the seabed depth profile is also changing as a result of beach erosion and sediment transport. The summer (non-eroded) sloping sandy beach profile is observed to be changing to the winter profile as eroded sediment is being deposited and accumulated offshore [4]. Sandy beaches are constantly being threatened by erosion, in response to waves, winds, storms and sea level rise [5]. Erosion is causing the sand particles to move alongshore and/or drift out to sea. As the loss of sand is an important factor, some beaches need physical intervention to deal with erosion, including shore hardening and beach nourishment.

Despite the ocean’s persistent encroachment, the coastline urban developments are the man-made permanent changing of beaches, where the coastlines are being dredged and at some places, reclaimed as artificial land [6]. Dumping landfill materials at the beaches is a common practice to protect marinas and other holiday resorts from further erosion. Dredging the seabed is another engineering practice to protect ports and harbors from sediment deposition.

Owing to the highly variable nature of the sea, we do not yet have a full understanding or description of the mixing and transport process of effluent discharge plumes from coastal plants outfalls [1,7]. A far field modeling study is presented to investigate the effect of erosion on a sloping sandy seabed upon mixing and dispersion of the outfall effluent discharges, where the seabed depth profile is modelled as the power functions of distance from the beach [8,9,10]. The analytical solutions of a two-dimensional advection-diffusion equation with a point source is illustrated graphically by plotting contours of concentration to replicate the dispersion of effluent plumes discharged from a sea outfall. The maximum value of shoreline concentration is used as an environmental impact measure to evaluate how well the near-shore effluent discharge plumes are diluted in coastal waters [8,10]. This regulatory standard can therefore be specified as "does not exceed a certain level of concentration anywhere along the beach" to control public health risks in the areas where the coastal waters is used for recreational purposes.

The use of mathematical models has been a key strategy for the basis of sound engineering outfall design and for assessing the potential environmental impacts. In terms of the practical applicability, it is well recognized that the far field model can be applied as a tool to perform preliminary worst-case assessments. If this easy-to-use assessment indicates no far field impacts at all, no further action is needed and the use of more sophisticated and time-consuming three-dimensional hydrodynamic and water quality modeling can be avoided.

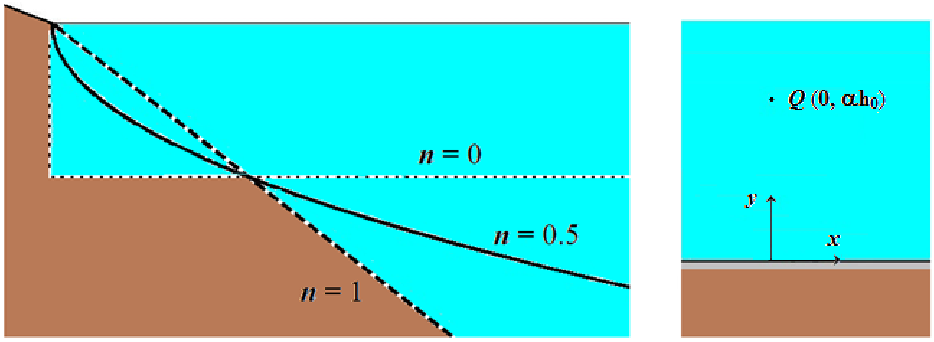

Fig. 1: Cross-section view of eroding sloping seabed (*left*); and plan view of single point source (*right*).

2. Model Formulation

Immediately after release from the outfall systems with multiport diffusers, vigorous and rapid mixing of the effluent stream is governed by the effluent buoyancy, momentum of the discharge and its interaction with the sea currents [1,2,7]. At the end of this mixing zone stage, adjacent positively buoyant effluent discharge plumes will interact with each other and merge to form a rising curtain, which then continues to drift away with the longshore currents [3,8,11]. In the far field and because of relatively shallow water depth, it is observed that the effluent discharge plumes are bent-over and spreading towards the shoreline [8,9,11].

As we are only concerned with the effect of seabed depth profile, in the far field modelling, the coastline is considered to be straight and the sea wide, and the effluent discharges plume is assumed to be vertically well-mixed over the water depth. For simplicity, we model the sloping seabed depth h (positive below the water surface) as the power function of distance y from the beach, $h = my^n$ $(0 < n \leq 1)$. As illustrated in Figure 1(*left*), in the limit as $n \to 0$, the seabed depth is constant $(h = m)$, known as a flat seabed, and when $n = 1$, it is known as the (non-eroded) sloping seabed with slope m. The seabed profiles due to bed erosion are represented the values of $n < 1$, which is usually deeper near the beach and relatively shallower offshore due to sediment deposition. The longshore current is assumed to be steady with a speed U_0 and remains in the x-direction parallel to the beach (positive to the right of discharge site) at all times. Other complexities such as tidal motions, density and temperature are also ignored. For shallow waters, dispersion in the vertical direction occurs much faster than in the lateral direction [8,9]. The dispersion processes are represented by dispersivity coefficient D_0, and since the effluent discharge plumes in a steady current become very elongated in the x-direction, dispersion process in the x-direction is neglected. A point source $(0, \alpha h_0)$ is used to refer to the location of the end of the outfall's pipe discharging effluent streams with a rate Q as shown in Figure 1(*right*).

The far field two-dimensional advection-diffusion equation for effluent discharges plume concentration $c(x, y)$ $c(x, y)$ from a point source is given by

$$\frac{\partial}{\partial x}(hUc) - \frac{\partial}{\partial y}\left(hD\frac{\partial c}{\partial y}\right) = Q\,\delta(x)\delta(y - \alpha h_0) \tag{1}$$

with the boundary conditions $hD\frac{\partial c}{\partial y}=0$ at the (coastline) beach $y=0$ and $c(x,y)\to 0$ as $y\to\infty$ (i.e., the concentration is ultimately dissolved at far distance distances), where h_0 is an arbitrary reference water depth.

In terms of dimensionless quantities $y=y_* h_0$, $x=x_* h_0$, $c(x,y)=c_*(x_*,y_*)Q/h_0{}^2U_0$, and by setting $h=m\,y_*{}^n\,h_0$, $U=U_0 y_*{}^{1/2}$ and $D=D_0 y_*{}^{3/2}$, we obtain an exact analytical solution, using the Laplace transform, in the form [12]

$$c_*(x_*,y_*)=\frac{\lambda}{m x_*}\left(\frac{1}{\alpha y_*}\right)^{\nu/2}\exp\left\{-\frac{\lambda(y_*+\alpha)}{x_*}\right\}I_\nu\left(\frac{2\lambda\sqrt{\alpha y_*}}{x_*}\right), \tag{2}$$

where the model parameter $\lambda=h_0U_0/D_0$ represents the effluent plume elongation in the *x*-direction, and I_ν is the modified Bessel functions of first kind of order $\nu=n+1/2$ [13,14].

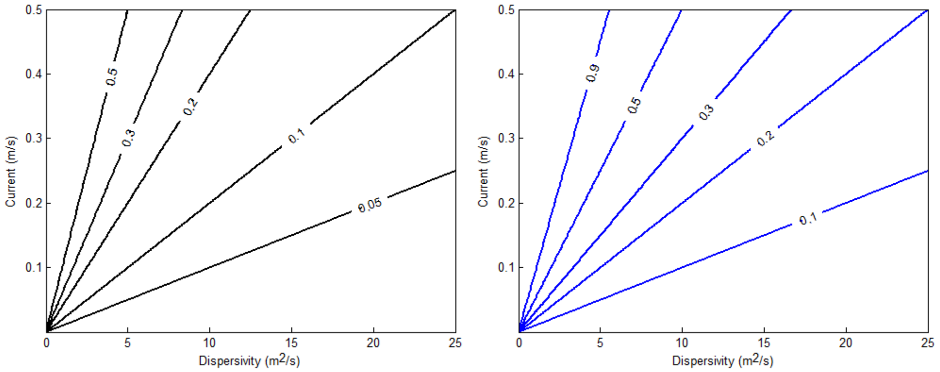

Fig. 2: Model parameter λ for water depth: $h_0=5$ m (*left*) and $h_0=10$ m (*right*).

To investigate the uncertainty in sea conditions, Figure 2 shows the graphs of λ for some relevant measured values of U_0 and D_0 for shallow water depth [8,15] for two values of $h_0=5$ m and $h_0=10$ m. The larger values of λ are mostly due to a stronger current U_0 with a smaller value of D_0 [8,9]. Thus, the values for λ in the range of 0.1 to 0.5 are suitable for a moderate current, and much smaller values of λ represents a calm sea condition. We note also that the variations in the *y*-direction of current U is proportional to $h_0{}^{1/2}$ and coefficient of dispersivity D to $h_0{}^{3/2}$. These scaling are appropriate for a turbulent shallow-water flow over a smooth bed [9,15,16].

For a shorter outfall, i.e., near-shore discharges, it is observed in the far field that the effluent discharge plumes are turning over and heading towards the coastline. Thus, the appropriate measure for assessing the impact of near-shore effluent discharges from a sea outfall would be the shoreline's concentration values. In the limit as $y_*\to 0$ and by replacing I_ν by its asymptotic form, we obtain, using the Gamma function $\Gamma(\nu)$, the concentration at the beach

$$c_*(x_*,0)\;\approx\;\frac{1}{m\,\Gamma(\nu+1)}\left(\frac{\lambda}{x_*}\right)^{n+3/2}\exp\left(-\frac{\lambda\alpha}{x_*}\right). \tag{3}$$

By differentiating with respect to x_*, the concentration at the beach has a maximum value of

$$c_n = \frac{1}{m}\frac{1}{\Gamma(\nu+1)}\left(\frac{n+3/2}{\alpha e}\right)^{n+3/2}, \tag{4}$$

which occurs at a downstream distance $x_n = \lambda\alpha/(n+3/2)$. The maximum value is inversely proportional to the slope m and to the point source length $\alpha^{n+3/2}$ provided that $\alpha > 5/2e$. This result shows and agrees with the standard practice that building a longer sea outfall can meet the standard criterion "does not exceed a certain level of concentration anywhere along the beach".

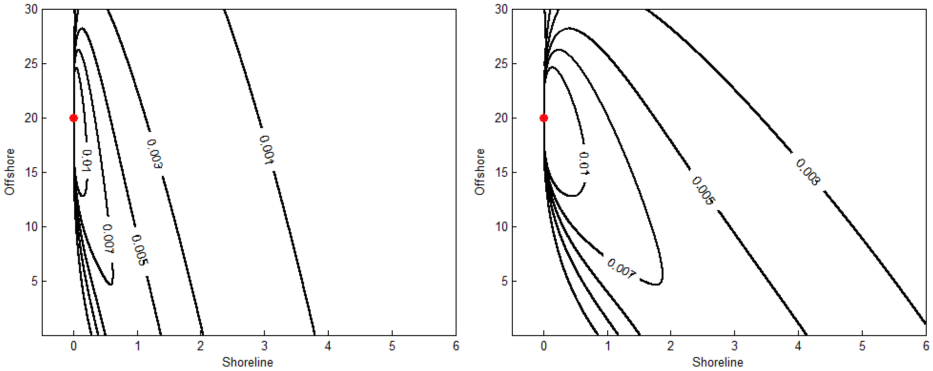

Fig. 3: Far field effluent plumes on non-eroding sloping seabed with: $\lambda = 0.1$ (*left*) and $\lambda = 0.3$ (*right*).

3. Effluent Discharges on Non-Eroding Sloping Seabed

As shown in Figure 1(*left*), when $n=1$ (i.e., $\nu = 3/2$), the depth profile is known as the (non-eroding) sloping seabed. Most sea outfalls are built predominantly on the sloping sandy beaches. The solution Eq. (2) simplifies to

$$c_*(x_*, y_*) = \frac{\lambda}{m x_*}\left(\frac{1}{\alpha y_*}\right)^{3/4} \exp\left\{-\frac{\lambda(y_* + \alpha)}{x_*}\right\} I_{3/2}\left(\frac{2\lambda\sqrt{\alpha y_*}}{x_*}\right). \tag{5}$$

For the model applications, the value of $m = 0.05$ will be used in all plots. For the near-shore discharges, the contours of Eq. (5) are plotted in Figure 3 for two different values $\lambda = 0.1$ and $\lambda = 0.3$ for a point source length $\alpha = 20$ (which corresponds, for a shallow water depth of $h_0 = 5$ m, to a shorter outfall length of 100 m). It is clear that the larger the values of λ, the spreading and transportion the effluent discharge plumes are to a further downstream distance. As the water depth is gradually decreasing towards the beach (i.e., the line $y = 0$), the plumes are heading and spreading towards the beach and cause a higher build up in concentration close to the beach.

Similarly when $n=1$, the concentration at the beach Eq. (3) reduces to

$$c_*(x_*, 0) \approx \frac{1}{m}\frac{4}{3\sqrt{\pi}}\left(\frac{\lambda}{x_*}\right)^{5/2} \exp\left(-\frac{\lambda\alpha}{x_*}\right). \tag{6}$$

As showed in Figure 4(*left*) for $\lambda = 0.2$, it has a maximum value that occurs at $x_1 = 2\lambda\alpha/5$ given by

$$c_1 = \frac{1}{m}\frac{4}{3\sqrt{\pi}}\left(\frac{5}{2\alpha e}\right)^{5/2} \approx \frac{0.61}{m\alpha^{5/2}}. \tag{7}$$

A maximum value of 0.0068 is obtained for $\alpha = 20$; this value is reduced by 43% to 0.0039 when the point source length is extended to $\alpha = 25$, and by more than 64% to 0.0025 when $\alpha = 30$. This result demonstrates that on a non-eroding sloping seabed, a longer sea outfall will guarantee minimal possible impact of effluent discharges in coastal waters [8,17].

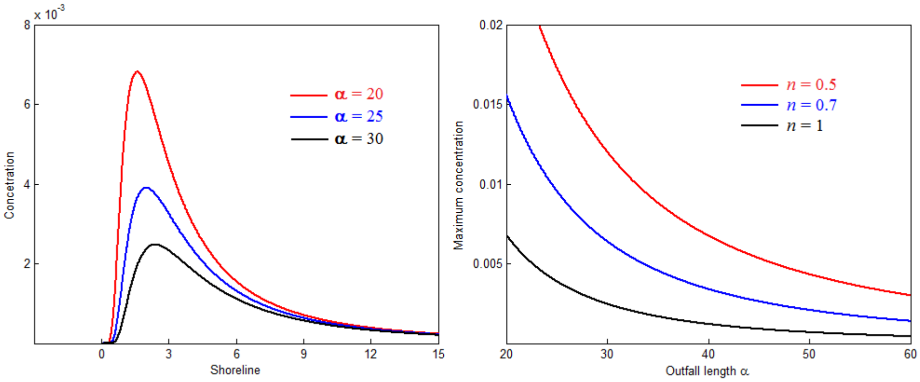

Fig. 4: Shoreline's concentration for $\lambda = 0.2$ (*left*), and maximum concentration values (*right*).

4. Effluent Discharges on Eroding Sloping Seabed

The contour plots of Eq. (2) for effluent discharges plume on an eroding sloping seabed is shown Figure 5 with $\lambda = 0.3$ and $\alpha = 30$ for two seabed depth profiles represented by $n = 0.5$ and $n = 0.7$. Due to large bed erosion (i.e., $n = 0.5$), the plume is wider and spreads over a larger downstream distance as relatively shallower offshore depth is associated with slower mixing and higher concentrations. This is due to the fact that in deeper water, the mixing is stronger as the current tends to be stronger, and there is a greater depth over which to disperse effluent plumes.

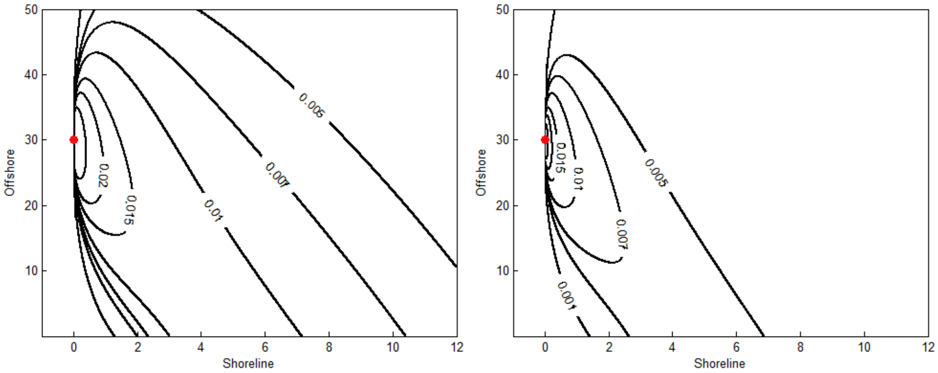

Fig. 5: Far field effluent plumes on eroding sloping seabed represented by: $n = 0.5$ (*left*) and $n = 0.7$ (*right*).

However, the effect of erosion on a sloping seabed, which represented by the values of $n < 1$, increases the maximum concentration values as shown in Figure 4(*right*). From Table 1, this increase persists and gets larger even for a longer point source length α. For example, a maximum value of 0.0012 is obtained for the length $\alpha = 40$ on a non-eroding sloping seabed with $n = 1$; this value is increased and is doubled to 0.0024 on an eroding sloping seabed represented by $n = 0.8$, and is quadrupled to 0.0048 when $n = 0.6$.

Similarly, as shown in Figure 6(*left*) for $\lambda = 0.3$, the bed erosion increases the shoreline's concentration level following effluent discharges from a point source length $\alpha = 30$. The maximum concentration value increases substantially due to seabed erosion, from a value 0.0025 on a non-eroding sloping seabed when $n = 1$ to 0.0047 on an eroded sloping seabed when $n=0.8$, and to 0.0088 on eroded sloping seabed when $n=0.6$. This result illustrates that a longer point source of more than $\alpha=30$ is needed to overcome the effect of erosion on a sloping seabed.

Table 1: Maximum concentration values for effluent discharges on eroding sloping seabed.

α	$c_1\times10^{-3}$	$c_{0.9}\times10^{-3}$	$c_{0.8}\times10^{-3}$	$c_{0.7}\times10^{-3}$	$c_{0.6}\times10^{-3}$
20	6.8	9.0	11.9	15.7	20.6
25	3.9	5.3	7.1	9.6	12.9
30	2.5	3.4	4.7	6.4	8.8
35	1.7	2.4	3.3	4.6	6.4
40	1.2	1.7	2.4	3.4	4.8
45	0.9	1.3	1.8	2.6	3.8
50	0.7	1.0	1.4	2.1	3.0

Figure 6(*right*) shows the relationship between maximum value of concentration at the beach and the sloping seabed depth profile exponent *n* for three values of the point source length α. Again, the result indicates that a longer sea outfall is required to sustain the change in a sloping seabed depth profile due to erosion. However as shown in Figure 7(*left*) for c_1/c_n, in comparison with the non-eroding sloping seabed as represented by $n=1$, even for a long point source with $\alpha=50$ (dotted curve), the maximum value is more than double that of c_1 for relatively small bed erosion represented by $n>0.7$. This suggests extending the outfall length alone may not be enough to overcome the bed erosion. Another engineering practice to reduce the potential impacts is to install a multiport diffuser at the end of the long outfall pipe consisting of many closely spaced ports (or nozzles) designed to discharge a series of effluent streams [1,3].

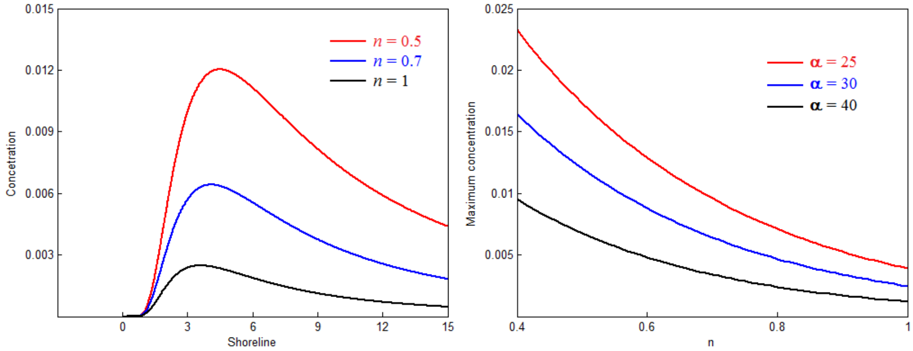

Fig. 6: Shoreline's concentration for $\lambda=0.3$ (*left*), and maximum concentration values (*right*).

For the quantitative example of a (perpendicular) line diffuser design, where a *p*-port diffuser is placed (in the *y*-direction) at the end of outfall long pipe [18], the maximum value of compounded concentration at the beach is approximated by

$$\frac{c_{dif}}{c_n}\approx1-\frac{p}{2}\left(n+\frac{3}{2}\right)\frac{\ell}{\alpha}, \tag{8}$$

where ℓ is the port (offshore) separation distance (small compared to α), and $(p-1)\ell/\alpha$ the (total) length of the line diffuser. For a particular case of discharging effluents from a 12-port diffuser with $\ell/\alpha=0.015$ on an eroding sloping seabed represented by $n=0.9$, we obtain $c_{dif}/c_{0.9}\approx0.8$. Similarly, for a 18-port diffuser on an eroding sloping seabed with $n=0.8$, we obtain $c_{dif}/c_{0.8}\approx0.7$. As shown in Figure 7(*right*), increasing the number of the ports will make the maximum value for the multiport diffuser with $\ell/\alpha = 0.015$ smaller than that of the single (port) outfall. This is because the individual effluent discharge plumes from each port are merged and swept by the current.

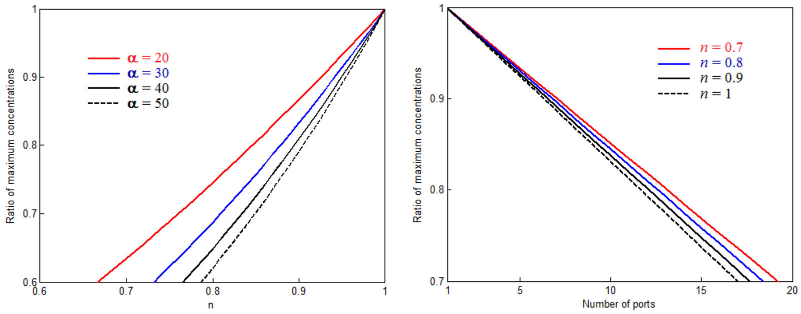

Fig. 7: Ratio of maximum concentrations: c_1/c_n (*left*), and c_{dif}/c_n (*right*).

5. Conclusion

Many parts of the coastline are constantly developed, mostly through dredging and being reclaimed as coastal artificial land. Dumping landfill materials at the beaches is a common practice to protect marinas and other holiday resorts from further erosion. Dredging the seabed is another engineering practice to protect ports and harbors from sediment deposition. Sea level rise also exacerbates beach erosion. Therefore, the bed erosion should be taken into consideration for assessing the environment impact of effluent discharges from a sea outfall.

Using a simple model of a changing seabed depth profile as the power functions of distance from the beach, and based on the maximum shoreline concentration values, it is found that building a longer sea outfall will sustain the erosion of a sloping seabed. Most modern marine outfall systems are well designed and installed with multiport diffusers, the additional effects of interaction and merging of effluent plumes from these multiple point sources are also found to reduce the maximum compounded value of concentration at the beach.

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Nomenclature

c	concentration of the (conservative tracer) effluent discharge
c_*	nondimensionalized concentration ($= c\, {h_0}^2 U_0 / Q$)
c_{dif}	maximum value of concentration at the shore for discharges from a multiport diffuser
c_n	maximum value of concentration at the shore for a given value of n
D	dispersivity coefficient in the y-direction (proportional to ${h_0}^{3/2}$)
D_0	a reference value of the dispersivity coefficient
h	water depth (positive below the surface)
h_0	a reference value of the water depth
I_ν	modified Bessel function of the first kind of order $\nu = n + 1/2$
ℓ	port separation distance of a multiport diffuser
m	slope of (non-eroding) seabed
n	exponent ($0 < n \le 1$)
p	total number of ports of a multiport diffuser
Q	a reference effluent discharge rate
U	speed of the longshore current in the x-direction (proportional to ${h_0}^{1/2}$)
U_0	a reference value of the current speed
x	longitudinal coordinate along the horizontal axis that is parallel to the shoreline
x_n	location where the maximum value of concentration is attained for a given value of n
x_*	nondimensionalized x-coordinate ($= x/h_0$)
y	lateral coordinate along the horizontal axis that is perpendicular to the shoreline
y_*	nondimensionalized y-coordinate ($= y/h_0$)
α	length of the (single) outfall from the shore
δ	Dirac delta function
λ	a model parameter ($= h_0 U_0 / D_0$)
ν	the order of Bessel function ($= n + 1/2$)
Γ	Gamma function